Les Houches Summer School “Quantum Optomechanics and Nanomechanics” August 2015

Casimir force : Optomechanics in quantum vacuum

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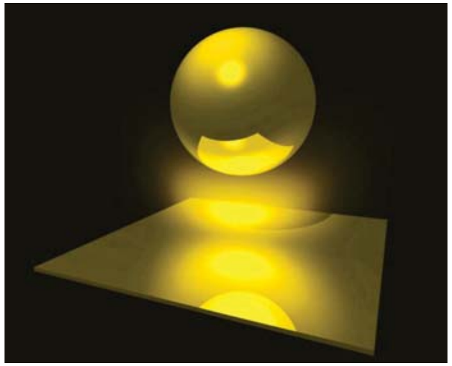
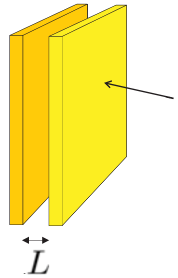


The Casimir force : ideal case

A universal effect from confinement of vacuum energy, which depends only on *ħ*, *c*, and geometry

$$E_{\rm Cas} = -\frac{\hbar c \pi^2 A}{720 L^3}\ , \quad F_{\rm Cas} = -\frac{{\rm d}E_{\rm Cas}}{{\rm d}L}$$



- Written here in an ideal case
  - Perfectly parallel plane mirrors
  - Perfectly reflecting mirrors
  - Zero temperature
- Attractive force (negative pressure)

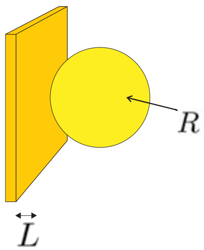
$$F_{\rm Cas} = P_{\rm Cas} A\ , \quad P_{\rm Cas} = -\frac{\hbar c \pi^2}{240 L^4}$$

$|P_{\rm Cas}| \sim 1{\rm mPa}$ at $L = 1\mu{\rm m}$

H.B.G. Casimir, Proc. K. Ned. Akad. Wet. (Phys.) **51** (1948) 79

The Casimir force : real case

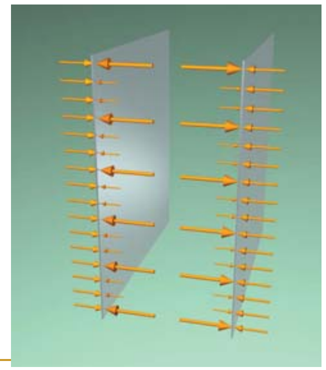
- Experiments performed with metallic plates (Gold)
  - Force depends on non universal properties of the real material plates used in the experiments
- Experiments performed at room temperature
  - Effect of thermal and vacuum field fluctuations to be taken into account
- Effect of geometry
  - Most precise experiments performed in the plane-sphere geometry
- Non ideal surfaces
  - Roughness, electrostatic patches, contamination …



Papers in *“Casimir Physics”* Lecture Notes in Physics **834** (Springer-Verlag, 2011)

Radiation pressure of quantum fluctuations

- « Optomechanics in Quantum Vacuum »
  - Quantum field fluctuations (vacuum and thermal fluctuations) pervading empty space → radiation pressure on mirrors
  - Force = difference of radiation pressures on inner and outer sides, summed up over all field modes
- « Scattering theory »
  - Mirrors characterized by scattering amplitudes depending on frequency, incidence, polarization
  - Transparent at high frequencies
  - Gives force for real mirrors
  - Can be extended to other geometries



M.-T. Jaekel & S. Reynaud, J. Physique **I 1** (1991) 1395

A short history of quantum vacuum

- 1911 : Introduction of zero-point fluctuations (*zpf*) by Planck

$$\overline{E} = \overline{n}_\omega \hbar\omega + \frac{1}{2}\hbar\omega \quad , \qquad \overline{n}_\omega = \frac{1}{e^{\hbar\omega/k_B T} - 1}$$

- 1913 : First correct demonstration of *zpf* (Einstein and Stern) : correct classical limit for quantum theory at high temperature
- Early successes before the advent of full quantum theory
  - ✓ 1914 : Prediction of effects of *zpf* on X-ray diffraction by Debye
  - ✓ 1924 : Observation of effects of *zpf* in vibration spectra by Mulliken
- 1916 : Vacuum energy puzzle discovered by Nernst
  - *zpf* of electromagnetic fields persist at zero temperature → vacuum field fluctuations in empty space
  - vacuum energy gives rise to a major problem at the interface with gravitation theory, still unsolved today

P.W. Milonni and M.L. Shih, Am. J. Phys. **59** 684 (1991)

Vacuum does not gravitate in the usual way

From a conservative estimation …

Bound on mean energy density in solar system → / Cutoff at the energy in accelerators (TeV) →

$$\frac{\rho^{\text{observ}}}{\rho^{\text{calcul}}} \lesssim 10^{-40}$$

… to the largest discrepancy ever seen between theory and experiment !

Now measured cosmic vacuum energy density (dark energy) → / Cutoff at the Planck energy →

$$\frac{\rho^{\text{observ}}}{\rho^{\text{calcul}}} \sim 10^{-120}$$

R.J. Adler, B. Casey and O.C. Jacob, Am. J. Phys. **63** (1995) 620

- Unsolved problem leading to many ideas, for example :
  - if the cutoff is set to fit the measured cosmic vacuum energy density, a length scale λ_{DE}=85µm is found - gravity could be affected below λ_{DE}

E. G. Adelberger et al, Progress in Particle and Nuclear Physics **62** (2009) 102

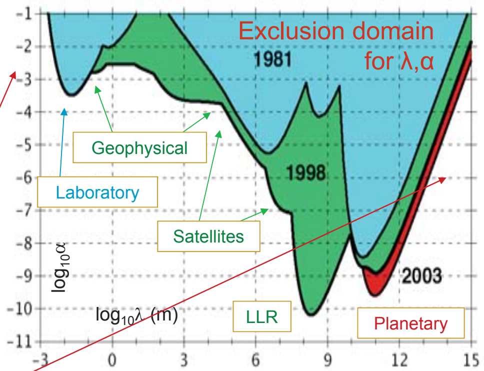
Search for scale dependent modifications of the gravity force law

- Exclusion plot for deviations with a Yukawa form

$$V(r) = -\frac{GMm}{r}\left(1 + \alpha e^{-\frac{r}{\lambda}}\right)$$

Windows remain open for deviations at short ranges $\lambda < 1\,\text{mm}$

or long ranges $\lambda > 10^{16}\,\text{m}$



Courtesy : J. Coy, E. Fischbach, R. Hellings, C. Talmadge & E. M. Standish (2003)

M.T. Jaekel & S. Reynaud Int. J. Modern Phys. **A 20** (2005) 2294

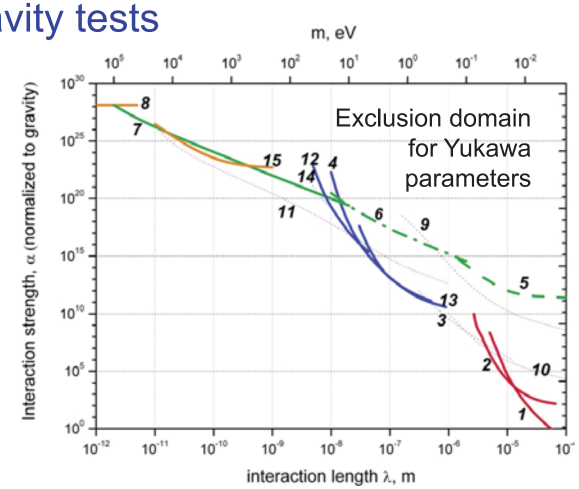
Short range gravity tests

- Gravity tests with torsion pendula (Adelberger's group)

  $\alpha = 1 \;\rightarrow\; \lambda < 56\mu m$

  95% confidence level

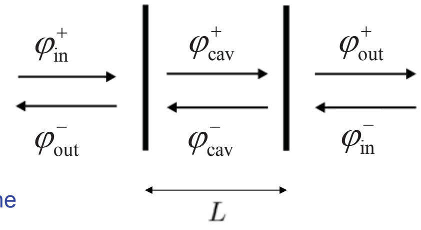
- Pushing the tests to smaller distances
  - Casimir experiments
  - Neutron experiments
  - Atomic physics



Recent review I. Antoniadis, S. Baessler, M. Büchner, V. Fedorov, S. Hoedl, A. Lambrecht, V. Nesvizhevsky, G. Pignol, K. Protasov, S. Reynaud, Yu. Sobolev, *Short-range fundamental forces* C. R. Phys. (2011) doi:10.1016/j.crhy.2011.05.004

A simple derivation of the Casimir effect

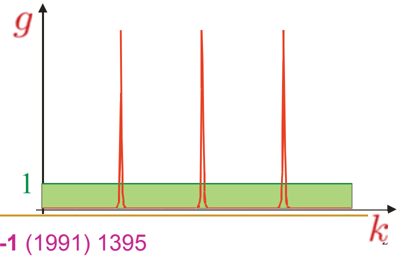
- Quantum field theory in 1d space
  - Counterpropagating scalar fields
  - Point-like mirrors



- Fabry-Perot cavity
  - Outer energies are the same as in the absence of the cavity
  - Inner energies are increased for resonant modes, decreased for non-resonant modes (⇔ Cavity QED)
- Energy enhancement factor

$$g = \frac{1 - \left|re^{2ikL}\right|^2}{\left|1 - re^{2ikL}\right|^2} \ , \quad r \equiv r_1 r_2$$

$r_{1,2}$: reflection amplitudes on mirrors



M.-T. Jaekel & S. Reynaud, J. Physique **I-1** (1991) 1395

A simple derivation of the Casimir effect

- The Casimir force is the sum over all field modes of the difference between inner and outer radiation pressures

$$F = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi c} 2\hbar\omega N_\omega \left(g(\omega) - 1\right)$$

Cavity confinement effect

Field fluctuation energy in the counter-propagating modes at frequency ω

Planck law including vacuum contribution

$$2\hbar\omega N_\omega = 2\hbar\omega\left(\overline{n}_\omega + \frac{1}{2}\right) = \frac{\hbar\omega}{\tanh\frac{\hbar\omega}{2k_B T}} \qquad \overline{n}_\omega = \frac{1}{e^{\hbar\omega/k_B T} - 1}$$

- The Casimir force can also be written in terms of causal amplitudes

$$F = 2\mathrm{Re}\left(\mathcal{I}_r^+\right) \ , \ \mathcal{I}_r^+ = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi c} 2\hbar\omega N_\omega \, f(\omega) \ , \ f = \frac{re^{2ikL}}{1 - re^{2ikL}}$$

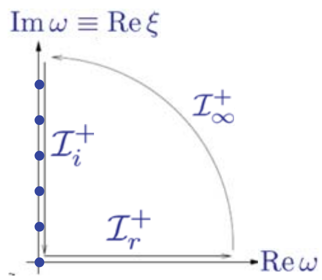
Using physical properties of mirrors

- The Casimir force can then be rewritten by using the known physical properties of the scattering amplitudes
- Causality : the loop function is analytic in the upper half of the complex plane $\mathrm{Im}\,\omega = \mathrm{Re}\,\xi \geq 0$

$\omega \equiv i\xi$

Cauchy theorem

$$\mathcal{I}_r^+ + \mathcal{I}_\infty^+ + \mathcal{I}_i^+ = 0$$



- High frequency transparency

$$\mathcal{I}_\infty^+ \to 0 \quad , \qquad \mathcal{I}_r^+ + \mathcal{I}_i^+ \to 0$$

- The force can be written as a sum over Matsubara frequencies (poles of N)

$$F = -2k_B T \sum_n{}' \frac{\xi_n}{c} f[i\xi_n]$$

$$\omega_n = i\xi_n \ , \ \xi_n = n\frac{2\pi k_B T}{\hbar}$$

$$\sum_n{}' f(i\xi_n) \equiv \frac{1}{2}f(0) + \sum_{n=1}^{\infty} f(i\xi_n)$$

Two plane mirrors in 3d space

- Electromagnetic fields in 3d space with parallel mirrors
  - Static and specular scattering preserves frequency ω, transverse wavevector $\mathbf{k}$, polarization p
  - reflection amplitudes depend on these quantum numbers
- Some elements in the derivation to be treated with great care
  - effect of dissipation and associated fluctuations
  - contribution of evanescent modes
  - analytical continuation arguments
- Casimir pressure obtained as a sum over Matsubara poles

$$P = -2k_B T \sum_p \int \frac{\mathrm{d}^2\mathbf{k}}{4\pi^2} \sum_n{}' \kappa_n \, f_\mathbf{k}^p[i\xi_n] \qquad \xi_n = n\frac{2\pi k_B T}{\hbar}$$

$$f_\mathbf{k}^p[i\xi_n] = \frac{r_\mathbf{k}^p[i\xi_n]e^{-2\kappa_n L}}{1 - r_\mathbf{k}^p[i\xi_n]e^{-2\kappa_n L}} \qquad \kappa_n = \sqrt{\mathbf{k}^2 + \frac{\xi_n^2}{c^2}}$$

Model for reflection amplitudes

- Simple model ➔ Lifshitz's formula for the Casimir force
  - bulk mirror (very thick slab)
  - local dielectric response function $\varepsilon[\omega]$
  - reflection amplitudes on each mirror given by Fresnel laws

$$r_1^{\mathrm{TE}}[\omega] = \frac{k_z - K_z}{k_z + K_z} \quad , \quad r_1^{\mathrm{TM}}[\omega] = \frac{K_z - \varepsilon k_z}{K_z + \varepsilon k_z}$$

$$K_z = \sqrt{\varepsilon \frac{\omega^2}{c^2} - k^2} \quad , \quad k_z = \sqrt{\frac{\omega^2}{c^2} - k^2}$$

K_z and k_z longitudinal wave-vector in matter and vacuum

$$r_1^{\mathrm{TE}}[i\xi] = \frac{\kappa - K}{\kappa + K} \quad , \quad r_1^{\mathrm{TM}}[i\xi] = \frac{K - \varepsilon[i\xi]\kappa}{K + \varepsilon[i\xi]\kappa}$$

$$K = \sqrt{\varepsilon[i\xi]\frac{\xi^2}{c^2} + k^2} \quad , \quad \kappa = \sqrt{\frac{\xi^2}{c^2} + k^2}$$

K and κ obtained after Wick rotation

E.M. Lifshitz, Sov. Phys. JETP **2** (1956) 73

I.E. Dzyaloshinskii, E.M. Lifshitz, L.P. Pitaevskii, Sov. Phys. Uspekhi **4** (1961) 153

Models for metallic mirrors

- Simple models for the dielectric function for metals
  - bound electrons (inter-band transitions, tables of optical data)
  - conduction electrons
    - determined by conductivity σ
  - Drude model for conductivity
    - plasma frequency ω_P
    - Drude relaxation parameter γ

$$\varepsilon[i\xi] = \bar{\varepsilon}[i\xi] + \frac{\sigma[i\xi]}{\xi}$$

(ε and σ divided by ε_0)

$$\sigma[i\xi] = \frac{\omega_P^2}{\xi + \gamma}$$

- Drude parameters related to the density of conduction electrons and to the static conductivity
  - finite conductivity σ_0 ⇔ non null γ

$$\omega_P^2 = \frac{nq^2}{\varepsilon_0 m^*}$$

$$\sigma_0 = \frac{\omega_P^2}{\gamma}$$

A. Lambrecht & S. Reynaud Eur. Phys. J. **D8** 309 (2000)

Calculations for metallic mirrors (room T)

- Pressure varies wrt ideal Casimir formula

M. Boström and B.E. Sernelius, Phys. Rev. Lett. **84** (2000) 4757

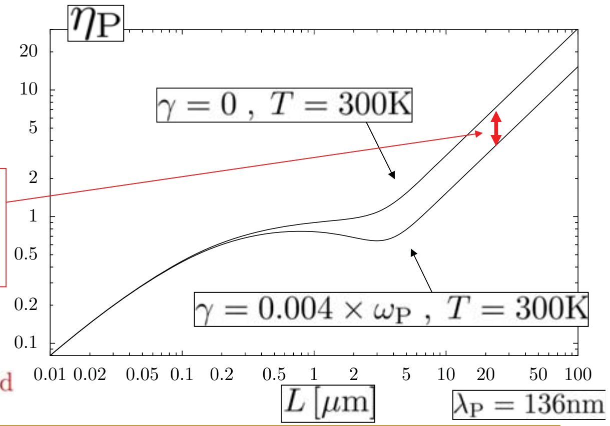
$$\eta_P = \frac{P}{P_{\mathrm{Cas}}}$$

$$P_{\mathrm{Cas}} = -\frac{\hbar c \pi^2}{240 L^4}$$

- small losses lead to a large factor 2 at large distances

$\bar{\varepsilon} = 1$ for this plot

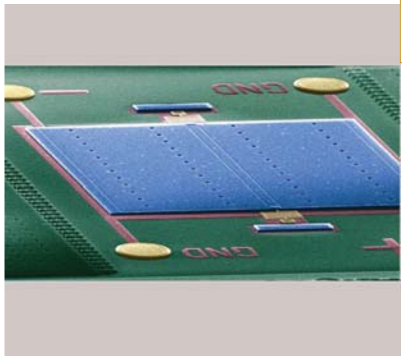
Drude parameters of Gold



G. Ingold, A. Lambrecht, S. Reynaud, Phys. Rev. **E80** (2009) 041113

Casimir experiments at IUPUI ..

Most precise and reliable experiments to date : dynamic measurements of the resonance frequency of a micro-torsion resonator



Courtesy Ricardo S. Decca (Indiana U – Purdue U Indianapolis)

Micro Torsion Oscillator :

500 μm × 500 μm × 3.5 μm

Torsion Equation

$$I\ddot{\theta} + \Gamma\dot{\theta} + K\theta = \tau(t)$$

τ(t) : acting torque

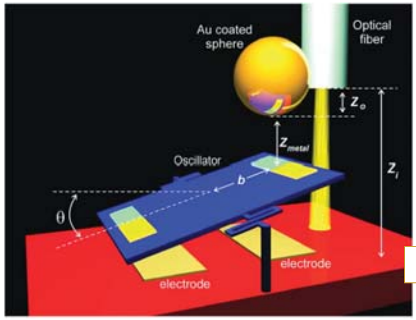
Free Oscillation Frequency

$$\omega_0^2 = \frac{K}{I} \quad , \quad \Gamma \ll \omega_0$$

→ Ricardo S. Decca's talk at Pan-American Advanced Study Institute "Frontiers of Casimir Physics", October 2012, Ushuaia, Argentina
http://physics.iupui.edu/graduate/program

.. Casimir experiments at IUPUI

Effect measured through the shift of the resonance when a gold-covered sphere approaches the gold-covered plane of the torsion plate



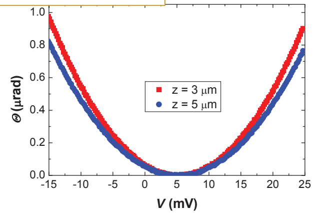
Sphere Radius: R = 150 μm

Distances: L = 0.16 - 0.75 μm << R

Frequency shift $\omega_r^2 = \omega_0^2 - \frac{b^2 G}{I}$

G = gradient of Casimir force $G = \frac{\partial F}{\partial L}$

Courtesy R.S. Decca, IUPUI



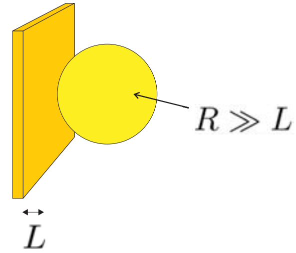
Electrostatic calibration to compensate unknown voltages and measure parameters ▶

Measurement of the separation between bodies (up to a global offset) using two-color interferometry

Proximity Force Approximation

- Force in the plane-sphere geometry computed using the PFA ("Proximity Force Approximation")
- The plane-plane pressure is integrated over the distribution of local inter-plate distance

$$F_{\mathrm{PFA}} = \int_L^\infty \mathrm{d}A\, P(L')$$



- For a plane and a large sphere

$$F_{\mathrm{PFA}} = 2\pi R \int_L^\infty \mathrm{d}L'\, P(L')$$

$$G_{\mathrm{PFA}} = \frac{\partial F_{\mathrm{PFA}}}{\partial L} = -2\pi R\, P(L)$$

- Shift of the resonance is proportional to the pressure P which would be measured in the plane-plane configuration

$$\omega_r^2 - \omega_0^2 = \frac{b^2}{I} 2\pi R\, P(L)$$

R.S. Decca, D. Lopez, E. Fischbach *et al*, Phys. Rev. **D75** (2007) 077101

IUPUI experiments vs theory ..

Purdue measurements agree with predictions from the plasma model but deviate from predictions when dissipation is accounted for

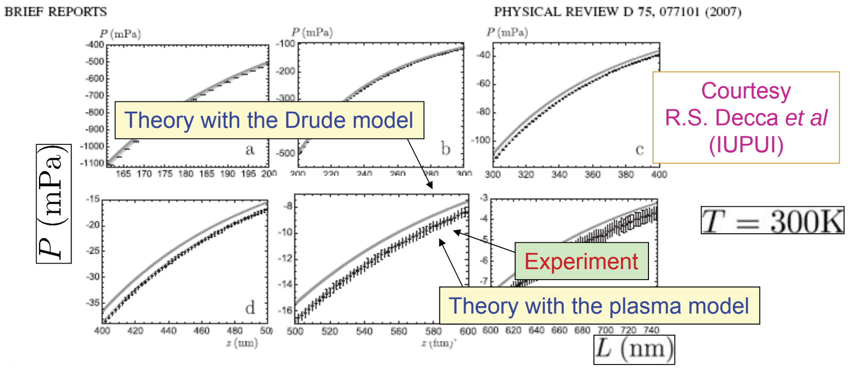


FIG. 1. Experimental data for the Casimir pressure as a function of separation z. Absolute errors are shown by black crosses in different separation regions (a–f). The light- and dark-gray bands represent the theoretical predictions of the impedance and Drude model approaches, respectively. The vertical width of the bands is equal to the theoretical error, and all crosses are shown in true scale.

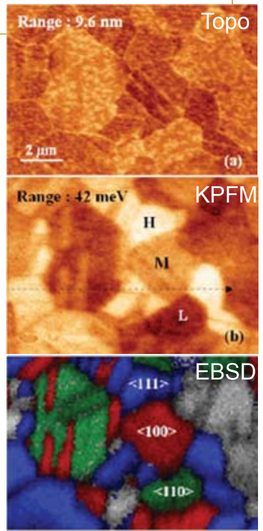
R.S. Decca, D. Lopez, E. Fischbach *et al*, Phys. Rev. **D75** (2007) 077101

.. IUPUI experiments vs theory

- Most precise and reliable experiment disagrees with the preferred theoretical model, but agrees with a poorer model !
- A list of suspects :
  - New forces ???
  - Artifacts in the experiments ??
  - Inaccuracies in the theoretical evaluations ?
    - A problem with vacuum energy ??
    - A problem with theoretical formula ??
    - The description of dissipation for metals ? ◀
  - Systematic effects misrepresented in the analysis ?
    - The contribution of plate roughness ? ◀
    - The contribution of electrostatic patches ? ◀ next slides
    - The corrections beyond PFA ? ◀
  - Something else ? ◀

The patch effect

- Surfaces of metallic plates are not equipotentials
  - Real surfaces are made of crystallites
  - Crystallites correspond to ≠ crystallographic orientations and ≠ work functions
- For ultraclean surfaces ▶
  (ultra-high vacuum, ultra-low temperature)
  - Patch pattern is related to topography
  - AFM, KPFM, EBSD maps are directly related
- Otherwise, contamination affects the patches
  - enlarges patch sizes and smoothes voltages
- Patch effect has been known for decades to be a limitation for precision measurements
  - Free fall of antiparticles, gravity tests, surface physics, experiments with cold atoms or ion traps...



N. Gaillard *et al*, APL **89** (2006)154101

Modeling the patches .. ..

- The pressure between two planes due to electrostatic patches can be computed by solving the Poisson equation

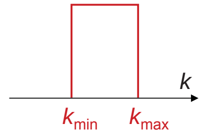
$$P = \frac{\varepsilon_0}{4\pi}\int_0^\infty \frac{\mathrm{d}k\ k^3}{\sinh^2(kL)}\left\{C_{11}[k] + C_{22}[k] - 2C_{12}[k]\cosh(kL)\right\}$$

- It depends on the spectra describing the correlations of the patch voltages
- The spectra had not been measured up to recently

$$C_{ij}[\mathbf{k}] = \int \mathrm{d}^2\mathbf{r}\, e^{-i\mathbf{k}\cdot\mathbf{r}}\, C_{ij}(\mathbf{r})$$

$$C_{ij}(\mathbf{r}) = \langle V_i(\mathbf{r})V_j(\mathbf{0})\rangle$$

- In the commonly used model, the spectrum was supposed to have sharp cutoffs at low and high-*k*, which is a very poor representation of the patches

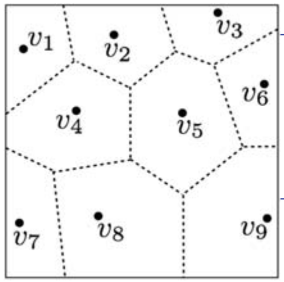


C.C. Speake & C. Trenkel PRL (2003) ; R.S. Decca *et al* (2005)

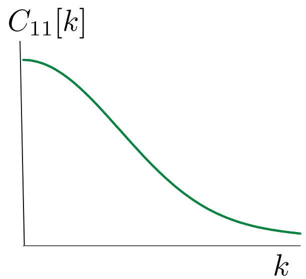
.. Modeling the patches ..

- A "quasi-local" representation of patches

R.O. Behunin, F. Intravaia, D.A.R. Dalvit, P.A. Maia Neto, S. Reynaud, Phys. Rev. **A85** (2012) 012504



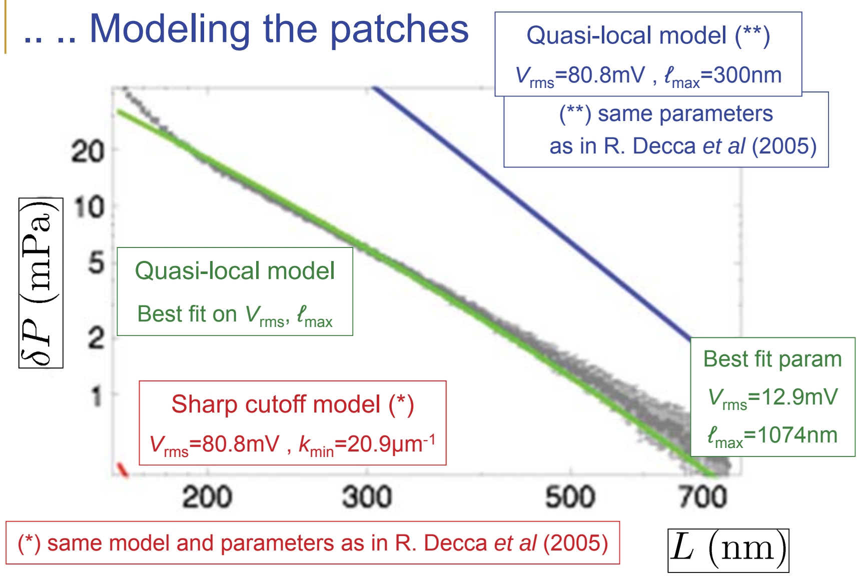
tessellation of sample surface and random assignment of the voltage on each patch



➧ This produces a smooth spectrum (no cutoff)

- Similar models used to study the effect of patches in ion traps
  - R. Dubessy, T. Coudreau, L. Guidoni, PRA **80** (2009) 031402
  - D.A. Hite, Y. Colombe, A.C. Wilson et al, PRL **109** (2012) 103001

.. .. Modeling the patches



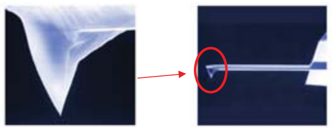
(*) same model and parameters as in R. Decca *et al* (2005)

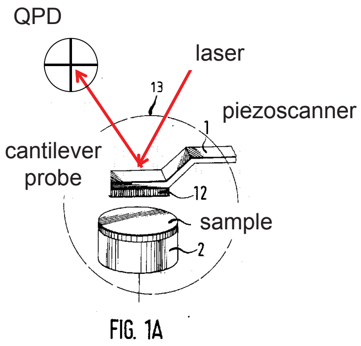
R.O. Behunin, F. Intravaia, D.A.R. Dalvit, P.A. Maia Neto & SR, PRA **85** (2012) 012504

Measuring the patches with KPFM ..

- Kelvin Probe Force Microscope (KPFM) = Kelvin Probe + Atomic Force Microscope setup
- KPFM measure local variations of work-function differences :
  - Electrostatic force dominant
  - Local measurement with a tip fixed at the end of a cantilever

Contact Electricity of Metals :
L. Kelvin, Philos. Mag. **46** (1898) 82



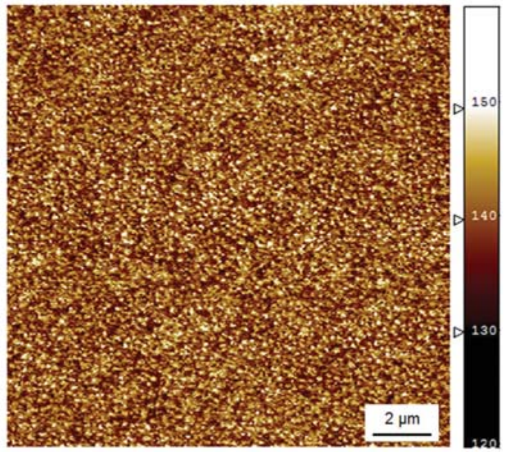


M. Nonnenmacher, M.P. Oboyle, and H.K. Wickramasinghe, APL **58** (1991) 2921

A. Liscio, V. Palermo, P. Samori, Accounts Chem. Res. **43** (2010) 541

.. Measuring the patches with KPFM

Electrostatic potential map $V_S(\mathbf{r})$



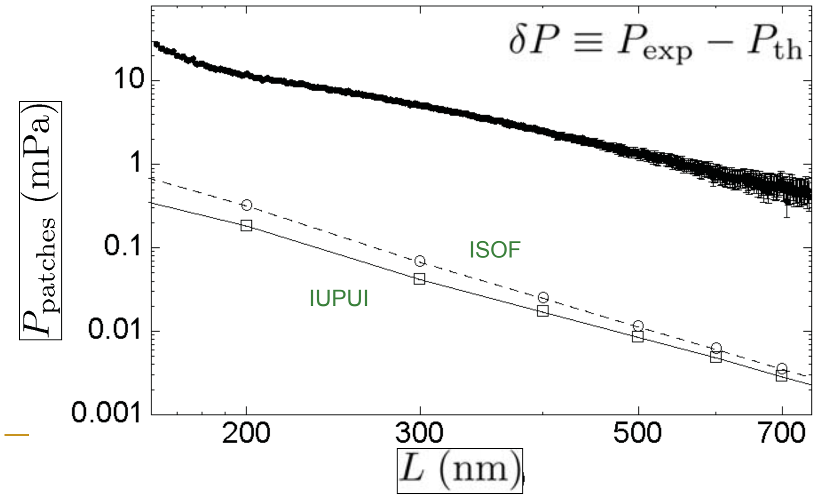
- Measurements performed by A. Liscio, ISOF Bologna
- Estimated resolution 160nm (Rayleigh criterion), should be sufficient for our purpose
- Samples provided by R.S. Decca (IUPUI), similar to those used in Casimir experiments
- Comparable results obtained in measurements by R.S. Decca at IUPUI

R.O. Behunin, D.A.R. Dalvit, R.S. Decca, C. Genet, I.W. Jung, A. Lambrecht, A. Liscio, D. Lopez, S. Reynaud, G. Schnoering, G. Voisin, Y. Zeng, Phys. Rev. **A90** (2014) 062115

Calculating the effect of patches

- Force evaluated in the plane-sphere geometry

R.O. Behunin, Y. Zeng, D.A.R. Dalvit, S. Reynaud, Phys. Rev. **A86** (2012) 052509



Conclusions at this point

- Casimir force tested at a ~5% accuracy
  - several sources of systematic effects investigated
  - smaller than the discrepancy δP which remains to be explained !
- Electrostatic patches measured on real samples
  - the spectrum looks like the one predicted by the quasi-local model
  - its magnitude and shape differ from those of the fitted spectrum which did explain the discrepancy δP
- More work needed in order to :
  - confirm the measurements of patches, with better resolution, larger scan sizes, also on spherical plates (plane plates measured up to now)
  - measure patches at the pressure of Casimir experiments (patch measurements done under normal pressure up to now)
  - push the investigations further and keep an eye on other suspects …

Thanks for your attention