Coupled spin-mechanical systems

Lecture 1

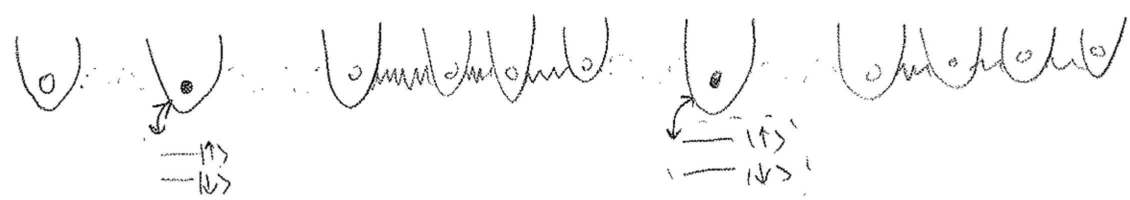
I. Intro and motivation

why would you want a spin-coupled mechanical system: <u>nonlinearity</u>

what can you do?

- generate and detect nonclassical states of motion
- mediate and transduce quantum information
- measure spin with mechanics and vice versa
- enhanced control over spin state
- enhanced spin sensitivity via spin squeezing

a futuristic idea: phononic quantum networks (Habraken et al NJP 14, 115004 (2012))



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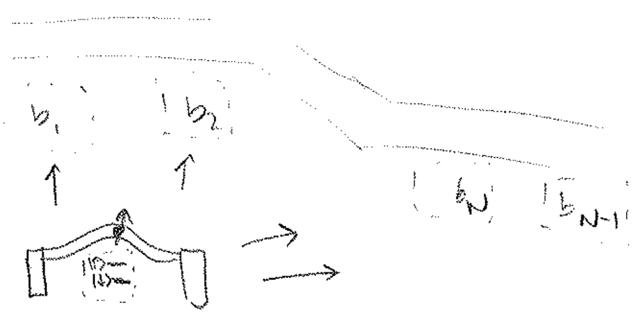


admittedly ... a bit far out

having a highly coherent & easy-to-control spin is very advantageous. the NV center in diamond is both

examples of other spin systems coupled to mechanical oscillators

- defects/donors in Si, SiO_2
- superconducting qubits
- GaAs quantum dots
- atoms (real ones)

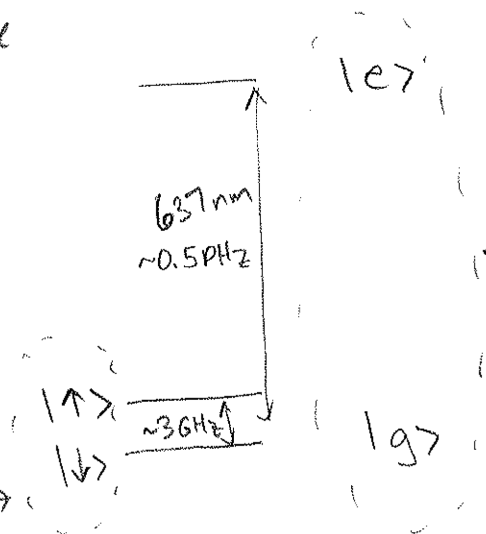
II. The nitrogen-vacancy center in diamond: general intro

• <u>solid-state</u> artificial <u>atom</u>

solid-state →
- easy to "trap"
- patternable
- easy to address individually
- compatible w/ on chip electronic/optical devices

atom →
- quantized energy levels
- optical transitions
- spin
- long coherence times

• two possible "spin" systems



← excited state "spin"

ground state "spin" →

plus more! nuclear spins
spin triplet ground state
6 excited state levels

- why does ground state spin coherence last up to several ms at room temperature and almost a second @ low T (4K)
  - → high Debye temperature
  - → deep center in a wide band gap material
  - → ^{12}C is nuclear spin free
  - → high elemental purity possible (hard to dope w/ most)
  - → small spin-orbit coupling
- many different ways of forming NVs
  - → naturally occurring
  - → implantation of nitrogen
  - → CVD growth with nitrogen "δ-doping"
- some examples of what has been achieved with NV's
  - → quantum memories at room temperature w/ 1-second coherence times (^{13}C nuclear spin)
  - → entanglement between NV spins 3 m apart
  - → sensing!
    - single electron spin external to diamond
    - almost single nuclear spin external to diamond and single nuclear spins inside diamond
    - vortices in superconductors, proteins
    - picometer-scale motion of magnetized mechanical resonator

· outstanding challenges facing NVs

→ deteriorated optical and spin properties near surfaces

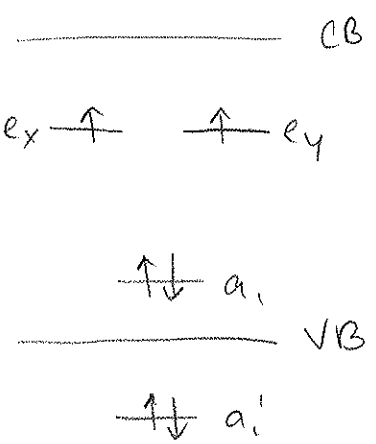
→ truly 1-nm scale position control

→ spectral diffusion of optical line

→ phonon side band

III. Nitrogen vacancy center: structure

NV^- is negatively charged NV. contains 6 electrons that occupy 2 A_1 (a_1, a_1') and a degenerate E (e_x, e_y) molecular orbitals



a_1, a_1', e_x, e_y are made up from the dangling bonds of the 3 carbon and nitrogen surrounding the vacancy: $\sigma_1, \sigma_2, \sigma_3, \sigma_N$

$$\left.\begin{aligned} e_x &= 2c_1 - c_2 - c_3 \\ e_y &= c_2 - c_3 \\ a_1 &= c_1 + c_2 + c_3 + \lambda n \end{aligned}\right\} \text{only mostly care about these}$$

$$a_1' = n - \lambda'(c_1 + c_2 + c_3)$$

(these linear combos derived from group theory considerations) some short words about group theory & introduction of concepts that will be useful later on when we talk about NV-strain interaction)

group theory can <u>qualitatively</u> predict the nature of the NV's interaction with field

- NV has $\underline{C_{3v}}$ symmetry

  the C_{3v} group has 6 elements : $\{E, C_3, C_3^2, \sigma_v, \sigma_v^2, \sigma_v^3\}$

- irreducible representations:

  fully symmetric under C_3 operations ← A_1 : 1-d IR fully symmetric under C_3 and other operations

  (1 ↓ 1-d)

  A_2 : 1-d IR, wave function picks up a sign under R_z rotations

  E : 2-d IR

- basis functions for the IR's.

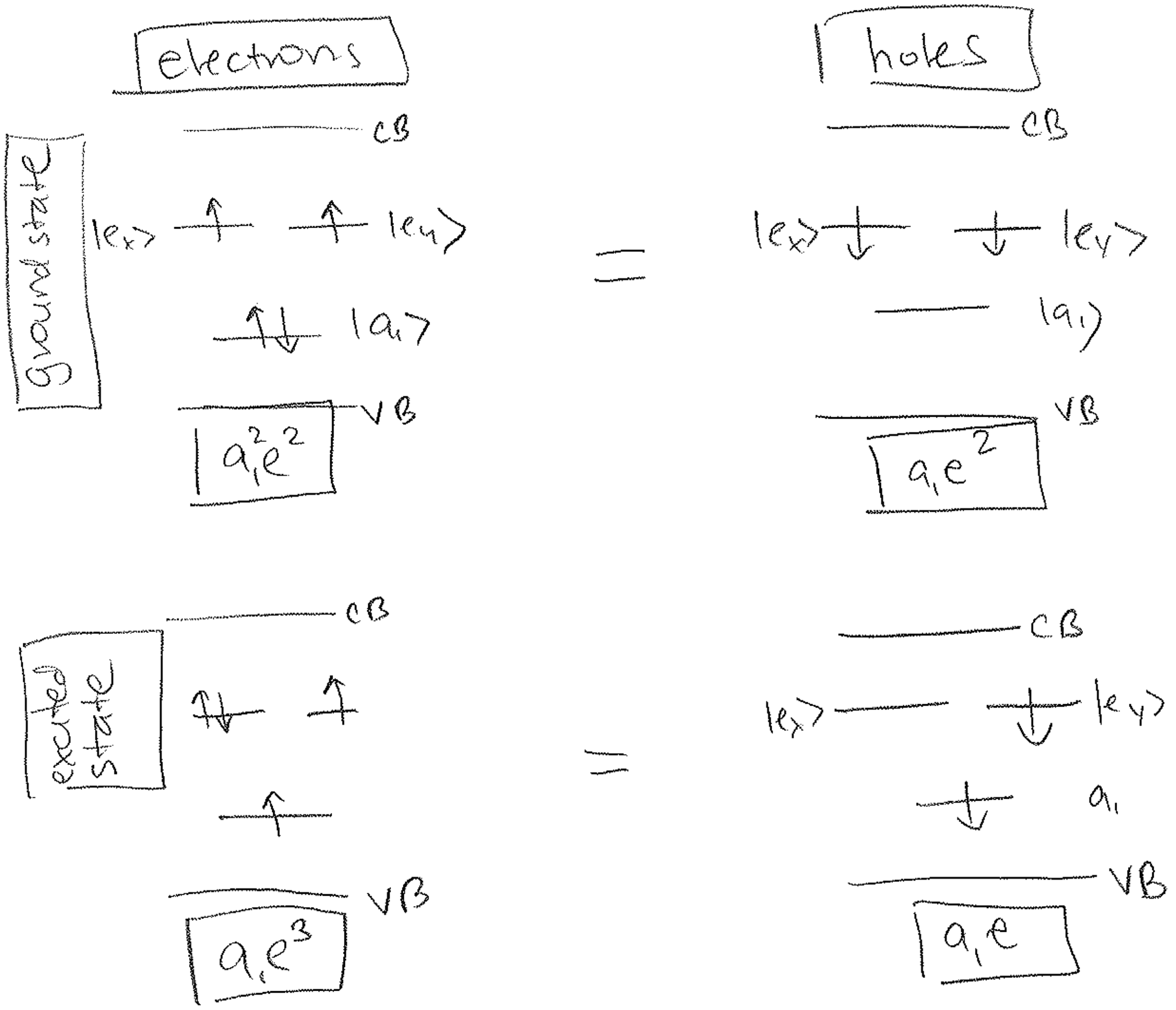
  A_1 : z $\quad z^2, x^2+y^2, R_z^2, R_x^2+R_y^2$

  E : $x, y, R_x R_y$ $\quad x^2-y^2, xy, xz, yz$

- e.g. z remains invariant under any symmetry operations

- a_1 transforms as A_1 and e_x, e_y orbitals transform as E

to simplify things, we can move from a 6-electron to a 2-hole picture (remember only treating $|a_1\rangle$ and $|e_x\rangle$ & $|e_y\rangle$ states



though picture above shows it, we have not yet formally considered spin

spin triplet favored (due to minimizing Coulomb interactions) using an antisymmetric orbital wavefunction

GS is a spin triplet, orbital singlet

ES (excited state): orbital doublet, spin triplet spin-spin, spin-orbit, strain, electric, magnetic fields all affect the ground and excited state energy levels:

<u>ground state</u>: $\mathcal{H}_{gs} = (D_{gs} + d^{\parallel}\Pi_{\parallel})S_z^2 + \gamma\vec{B}\cdot\hat{S} - d^{\perp}[\Pi_x(S_xS_y + S_yS_x) + \Pi_y(S_x^2 - S_y^2)]$

$\Pi_{\parallel} = E_{\parallel} + \sigma_{\parallel}$

$\Pi_x = E_x + \sigma_x$

$\Pi_y = E_y + \sigma_y$

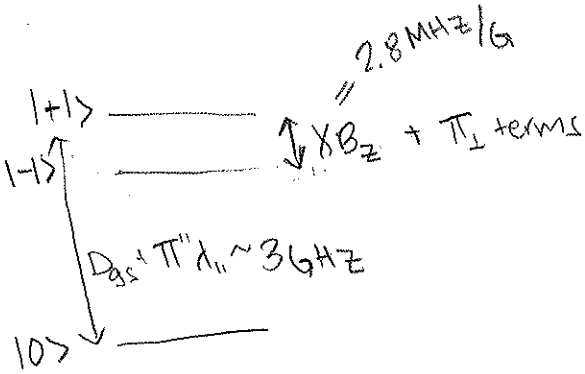
$\vec{E}_{\parallel}$: electric field along NV axis

$\sigma_{\parallel}$: strain "along" NV axis, made up of components that are A_1-symmetric (preserve symmetries of NV)

σ_x, σ_y: "perpendicular" strain components, made up of components that are E-symmetric, don't preserve symmetry of NV

of course electric field and strain are not exactly the same (strain is a tensor, electric field a vector) but we'll come back to this point using symmetry arguments later

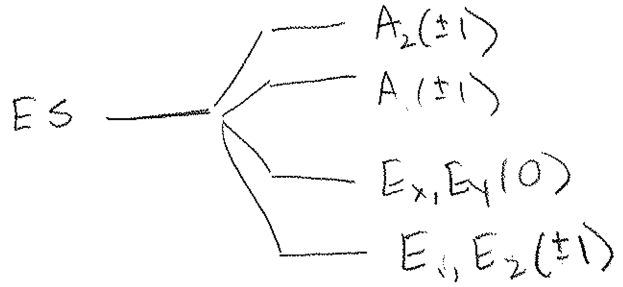
- D_{gs}: "zero-field splitting" arises from spin-spin interactions splits $|0\rangle$ from $|\pm1\rangle$
- $\gamma\vec{B}\cdot\hat{S}$: Zeeman splitting splits $|1\rangle$ from $|-1\rangle$
- $\Pi_{\parallel}$: shifts zero-field splitting
- $\Pi_{\perp}$: mixes $|\pm1\rangle$ states and splits resulting states



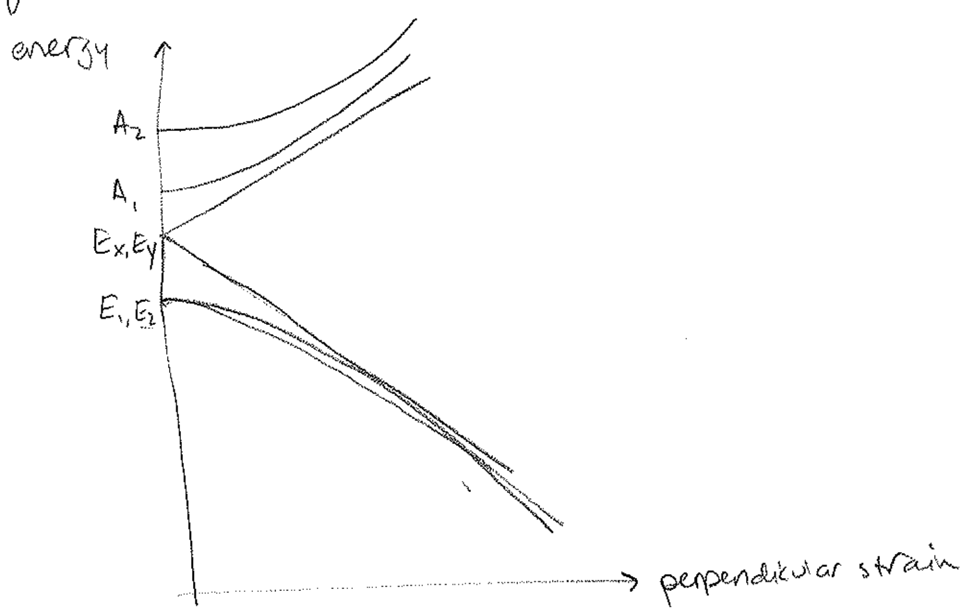
"spin" = $|0\rangle, |1\rangle$
or
$|0\rangle, |-1\rangle$
or
$|1\rangle, |-1\rangle$

excited state interactions:

→ no applied fields or strain: spin-spin and spin-orbit interactions split the spin triplet, orbital doublet into

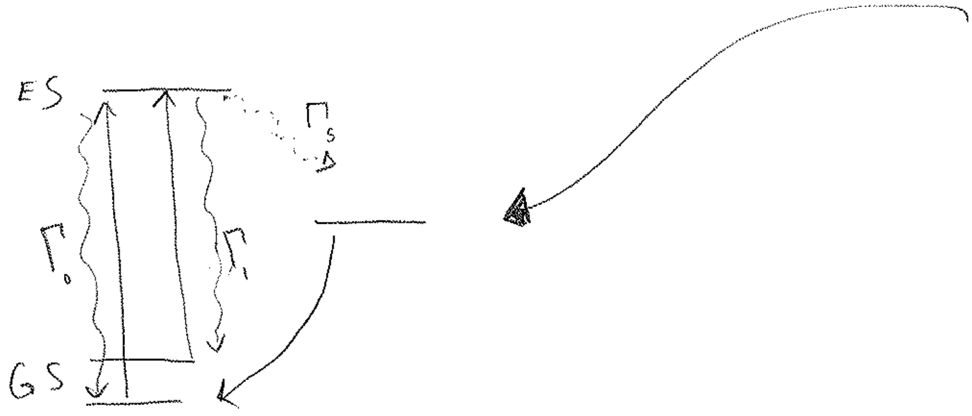


as a function of applied strain (or E field)



- E_x & E_y are particularly sensitive to strain because of orbital degeneracy
- excited state structure only resolvable at low ($\lesssim$ 10K) temperature due to phonon broadening
- richness of excited state structure allows for novel ways of quantum control of NV & mechanics

- another important structural element of NV are the <u>metastable singlet states</u>



$\rightarrow$ allows for optical readout and initialization of spin state

- ground state $|M_s=0\rangle$: cycling transition
  $|M_s=\pm1\rangle$: ~20% probability of triplet-singlet ISC (intersystem crossing)

- metastable singlet has long (~300ns) lifetime
  $\Rightarrow$ $|0\rangle$ state ~30% brighter than $|1\rangle$ state

- ISC mechanism not well understood
  in particular, the role of phonons

IV : The NV center: initialization, readout, and control

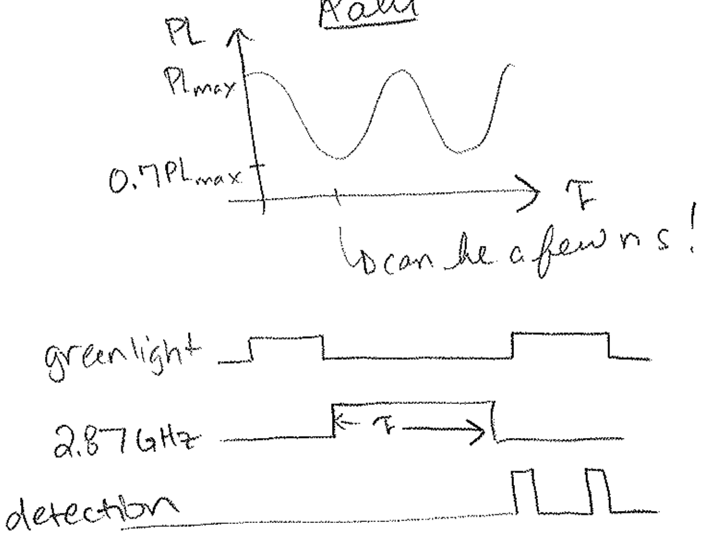
· initialization : shine green laser for ~500 ns to initialize into groundstate $|m_s=0\rangle$

· readout : again shine green laser light on NV & measure PL
$|0\rangle$ state 30% brighter than $|1\rangle$ state

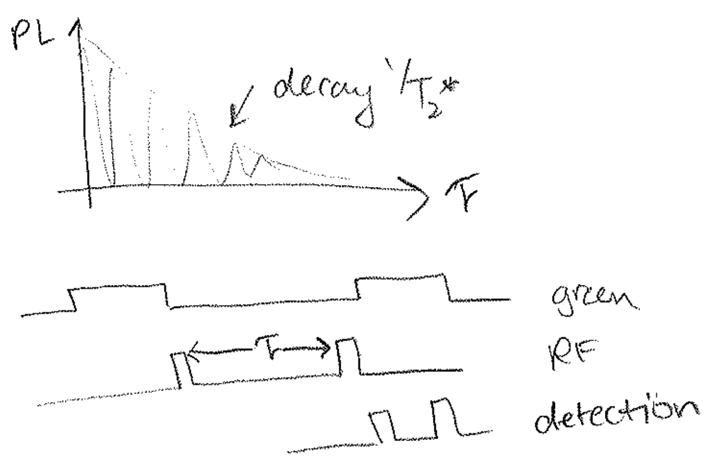
note: can also do resonant excitation @ low T for better "contrast" between $|0\rangle$ and $|1\rangle$ state PL

· control : microwave control of ground state spin

Rabi



Ramsey

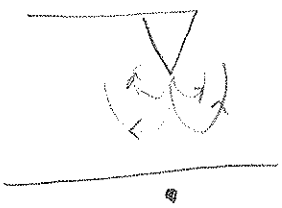


note: can also exert all optical control over ground state spin state (over whole Bloch sphere)

Now that we have basics of NV level structure and NV susceptibilities worked out, lets see how to couple mechanics and spins.

we have several choices of "spin" system and several choices of field coupling type. I will talk about 3 examples

① magnetic coupling

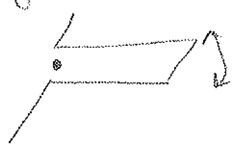


gs spin:

— $|+1,-1\rangle$

— $|0\rangle$

② strain coupling in ground state

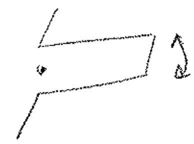


gs spin:

— $|+1\rangle$

— $|-1\rangle$

③ strain coupling to excited state



excited state "spin"

— $|E_x\rangle$

— $|E_y\rangle$

— $|0\rangle$

considerations: coupling strength g (want large energy shifts per quantum of motion)

spin decay: linewidth ($1/T_2^*$ in g.s. Γ_{NV} in e.s.)

mechanical decay: $\Gamma_{mech} = k_B T/\hbar Q$

for strong coupling want $g > \Gamma_n, \Gamma_{mech}$

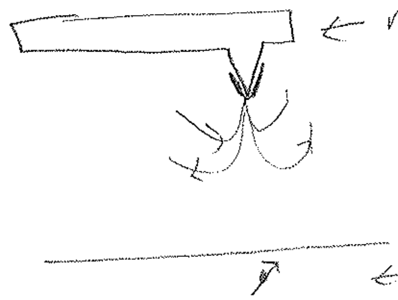
for high cooperativity want $C = g^2/\Gamma_{NV}\Gamma_{mech} > 1$

GS: long T_2^*, high B susceptibility

ES: short Γ_{NV}, high σ susceptibility

other considerations: effect of coupling modality on Γ_{NV} and Γ_{mech}

example 1: coupling GS spin to a magnetized cantilever



$$\mathcal{H} = \mathcal{H}_{NV,gs} + \mathcal{H}_{osc} + \mathcal{H}_{int}$$

$$\mathcal{H}_{NV,gs} = DS_z^2$$

$$\mathcal{H}_{osc} = \hbar\omega_m a^\dagger a$$

$$\mathcal{H}_{int} = \gamma \vec{B}_{AC} \cdot \hat{S}$$
$$= \hbar\gamma(\nabla B)a_0(a+a^\dagger)\sigma_z$$
$$= \hbar\lambda(a+a^\dagger)\sigma_z$$

$\boxed{\lambda = \gamma a_0 \nabla B}$ coupling strength: gives vibrational shift per energy quantum

how does λ compare to relevant system decay rates?

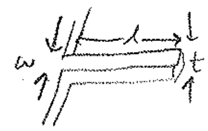
$$\Gamma_{NV} = 1/T_2^* \sim 1\text{-}10\text{ kHz}$$

$$\Gamma_{mech} = k_BT/\hbar Q = 10\text{ kHz} \quad @100\text{mK},\ Q = 2\cdot10^5$$

λ: $\gamma = 2.8\text{MHz}/G$, $\nabla B = 6\cdot10^6\text{ T/m}$ @ best (APL 100 013102 (2012) dysprosium tips)

$a_0 = \sqrt{\hbar/2m\omega_0}$ - depends on resonator geometry

assume singly clamped beam of dimensions



$$m = \rho(w\cdot t\cdot l)(1/4)$$
$$f_0 = .162\sqrt{E/\rho}\ t/l^2 \implies a_0 \sim \sqrt{\frac{l}{wt^2}}$$

you want to design beam to be long, narrow, and thin

200nm × 100nm × 20 μm $\Rightarrow q_0 = 1.8 \cdot 10^{-13}$ m

$\Rightarrow B_{AC} \cong 10^{6}$ T, $\boxed{\lambda = 28\,\text{kHz}}$

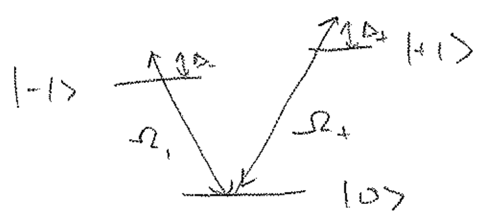
$$\boxed{\lambda > \Gamma_{NV}, \Gamma_{mech}\ !}$$

some notes about magnet choice and effects of thermomagnetic noise
- want small, single domain magnet!

some notes about surface-induced mechanical dissipation
- want small cross sectional area of resonator

- engineering a Jaynes-Cummings interaction b/w spin & resonator (Rabl et al, PRB 79, 041302 (2009))

working in GS manifold, simultaneously drive 0→1 and 0→−1 transitions in presence of a static B field



write $\mathcal{H}$ in frame rotating with the microwave frequencies Ω_- & Ω_+

$$\mathcal{H} = -\hbar\Delta_-|-1\rangle\langle -1| - \hbar\Delta_+|+1\rangle\langle +1| + \frac{\hbar\Omega_-}{2}\left(|0\rangle\langle -1| + |-1\rangle\langle 0|\right)$$
$$+\frac{\hbar\Omega_+}{2}\left(|0\rangle\langle 1| + |1\rangle\langle 0|\right)$$

now assume $\Omega_- = \Omega_+$ and $\Delta_- = \Delta_+$

$$\mathcal{H} = \sum_{i=-1,1} -\hbar\Delta|i\rangle\langle i| + \frac{\hbar\Omega}{2}\left(|0\rangle\langle i| + |i\rangle\langle 0|\right)$$

notice that $\mathcal{H}$ couples the state $|0\rangle$ to a "bright" superposition of $|+1\rangle$ and $|-1\rangle$: $|b\rangle = \frac{|-1\rangle + |1\rangle}{\sqrt{2}}$

and there is no coupling to $|d\rangle = \frac{|-1\rangle - |1\rangle}{\sqrt{2}}$ $\quad |d\rangle$: "dark state"

check: $\mathcal{H}|d\rangle = -\hbar\Delta|d\rangle$

$\mathcal{H}|0\rangle = \frac{\hbar\Omega}{\sqrt{2}}|b\rangle$

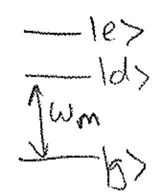
one can find the eigenstates of this Hamiltonian to be

$|d\rangle, |e\rangle = \cos\theta|b\rangle + \sin\theta|0\rangle, |g\rangle = \cos\theta|0\rangle - \sin\theta|b\rangle$

where $\tan(2\theta) = -\sqrt{2}\Omega/\Delta$

$\rightarrow$ (homework!!)

with eigenenergies $\omega_d = -\Delta, \omega_{e,g} = \frac{-\Delta \pm \sqrt{\Delta^2 + 2\Omega^2}}{2}$



when $\Delta < 0$, $|g\rangle$ is lowest-energy state

now write $\mathcal{H}$ in the $|e\rangle, |d\rangle, |g\rangle$ basis:

$$\mathcal{H} = \hbar\omega_m a^\dagger a + \hbar\omega_{eg}|e\rangle\langle e| + \hbar\omega_{dg}|d\rangle\langle d| + \hbar(\lambda_g|g\rangle\langle d| + \lambda_e|d\rangle\langle e| + H.c.)(a + a^\dagger)$$

$\lambda_g = -\lambda\sin\theta$

$\lambda_e = \lambda\cos\theta$

If we match ω_m to ω_{gd} and ω_{ed} is far detuned, then we can reduce the 3 level problem to an effective 2LS and going into interaction picture & making RWA

$$\mathcal{H}_{int}^{RWA} = \hbar(\lambda_g|g\rangle\langle d|a^\dagger + \lambda_g^*|d\rangle\langle g|a)$$

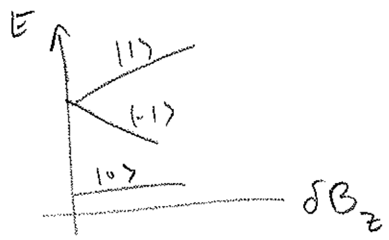
$$= \frac{\hbar\lambda_g}{2}[a\sigma^+ + a^\dagger\sigma^-] \quad \text{J-C Hamiltonian!}$$

but T_2^* near the surface (necessary to have high field gradients) usually not $>100\,\mu s$. plus, you have a big magnet right nearby.

this scheme allows to extend T_2^* through decoherence protection. how?

the bare states of the NV ($|+1\rangle$ and $|-1\rangle$) couple to a magnetic field linearly, and hence to magnetic fluctuations linearly

$\Rightarrow \Delta\omega_{01} = \gamma(\delta B_z)\cdot S_z$ random shifts of $0 \to 1$ qubit splitting cause line broadening

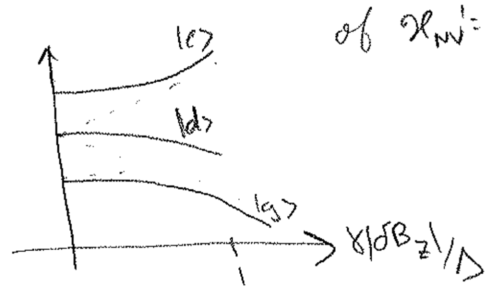


dressed state energies of $\mathcal{H}_{NV}' = \mathcal{H}_{NV} + \mathcal{H}_B$

$\downarrow$

fluctuati B field

but in the dressed state picture:



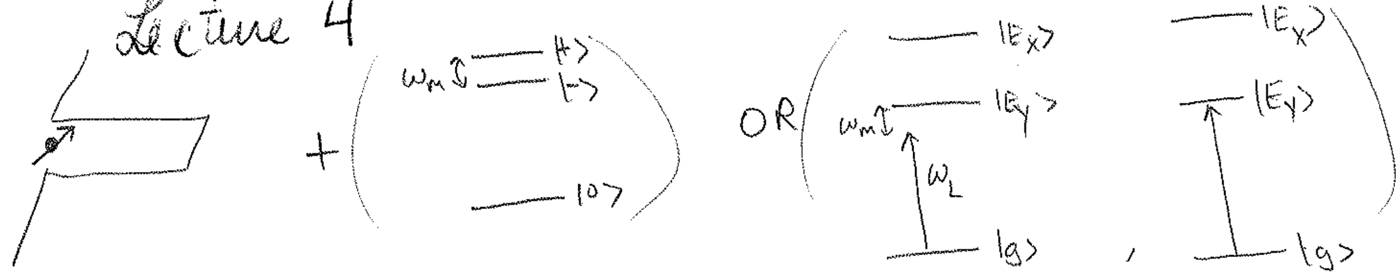
at small fields, $\Delta\omega_{dg}$ is insensitive to field fluctuations! (to 2nd order)

$\Delta\omega_{dg}^{(1)} = 0$

$$\Delta\omega_{dg}^{(2)} = \frac{(\gamma\delta B_z)^2}{\omega_{gd}}\left[1 - \tan^2\theta + \sin^2\theta\right]$$ quadratic shifts!

so, if decoherence is due to fluctuating B fields, then T_2^* will be greatly increased in this scheme

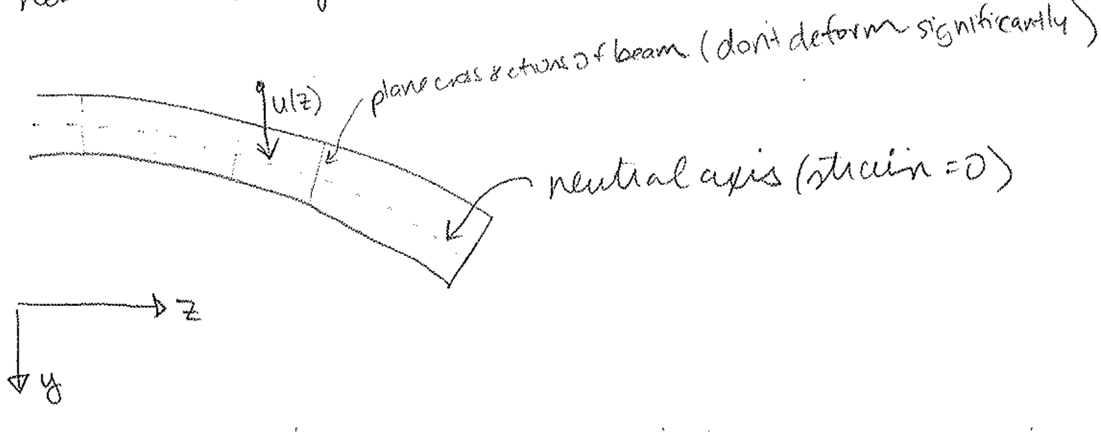
Lecture 4



we will discuss strain mediated coupling of spin to mechanics and specifically we'll address two schemes: one that utilizes ground state spin and one that utilizes excited state "spin"

But first, we need to know what kind of coupling strengths, λ, we can expect. for that we need to be able to calculate the strain in one of our designed beams, and then ultimately design our beam to maximize λ (again, always being mindful of the effect on cantilever Q and NV T_2^*). In this case, T_2^* will likely be limited by proximity to surface and nano fabrication-induced damage. The cantilever Q will be probably unpredictable, but we will want to minimize known origins of dissipation, like clamping loss.

Assume our mechanical structure is a singly (or doubly) clamped beam. We can apply Euler-Bernoulli theory: (simple scalar theory that works well for long, thin beams and small amplitudes of motion). assumption: plane cross sections of beam remain perpendicular to the neutral axis of the strained beam



under EB assumptions, the dynamic equation of motion is

$$EI \frac{\partial^2 u}{\partial z^4} = -\rho A \frac{\partial^2 u}{\partial t^2}$$

U: beam displacement in the y direction

E: elastic modulus

I: moment of inertia of the beam's cross section : $I = wt^3/12$

$$I = \int_A y^2 dA = \int_{-w/2}^{w/2} \int_{-t/2}^{t/2} y^2 dy dx = \frac{wt^3}{12}$$

$\leftarrow w \rightarrow$ $\updownarrow t$ y x

ρ: mass density

A: cross sectional area $(w * t)$

we can solve for $u(z,t)$ by imposing appropriate boundary conditions

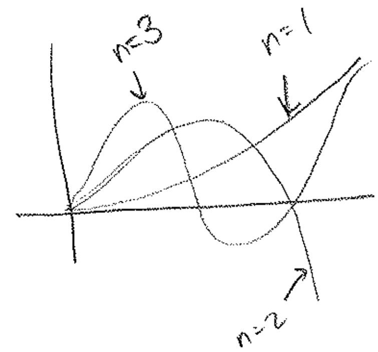
$$u(0) = u'(0) = u''(L) = u'''(L) = 0$$

$$\Rightarrow U_n(t,z) = u_n(z) e^{-i\omega_n t}$$

$$u_n(z) = a_n (\cos(\beta_n z) - \cosh(\beta_n z)) + b_n(\sin(\beta_n z) - \sinh(\beta_n z))$$

mode shapes:

$$a_n/b_n = -1.362, -0.98, -1.008, -1.000$$



β_n satisfies $\cos(\beta_n L)\cosh(\beta_n L) = -1$

$$\beta_1 L = 1.875, \beta_2 L = 4.69 \text{ etc}$$

eigenfrequencies: $\omega_n = \beta_n^2 \sqrt{\frac{EI}{\rho A}} = \underbrace{(\beta_n L)^2}_{\text{constants}} \sqrt{\frac{E}{12\rho}}\, t/L^2$

important: need to normalize the amplitude of the mode:

set value of $u_1(L) = x_0 = \sqrt{\frac{\hbar}{2 m_{eff} \omega_1}}$ where $m_{eff} = \frac{1}{2} L t w \rho$

would get same result by integrating the strain energy density $\left(\frac{E}{2}\right)\left(\frac{du}{dz}\right)^2$
over the volume of the cantilever and equating it to

$$\frac{1}{2} E_{zpm} = \frac{1}{4}\hbar\omega_0$$

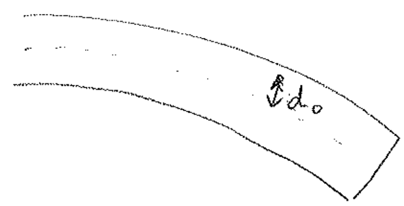
this technique is useful if you don't know effective mass

for a doubly clamped beam, analysis is identical & you end up
with a different effective mass ($0.4\,wtL\rho$) and different

β_n: $\beta_1 L = 4.73$
$\Rightarrow$ frequencies are higher

lets calculate strain now that we know modeshapes and their
normalized amplitudes:

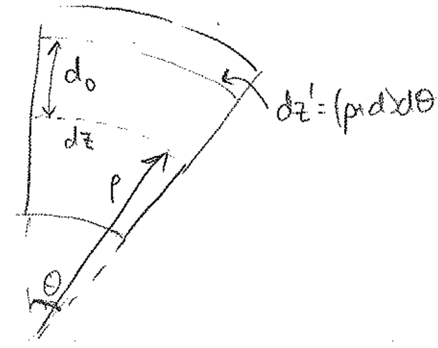
strain: $\epsilon = d_0 \dfrac{\partial^2 u}{\partial z^2}$



d_0: distance from neutral axis
to point of interest
$\Rightarrow$ positive strain is tensile strain at top of beam
negative strain is compressive strain at bottom of beam

EB beam (small deflections)
lets look at a small element of length dz along neutral axis
in unstrained case. for small deflections the length
of dz doesn't change but its curvature does & it now
forms a small arc ~~of~~ section of a circle of radius ρ where $dz = \rho\, d\theta$

lets look at section dz' at some distance
d_0 from neutral axis



$$dz' = (\rho + d)d\theta = \rho\, d\theta + d_0\, d\theta$$
$$= dz + d_0\, d\theta$$

strain $= \dfrac{dz' - dz}{dz} = \dfrac{d_0\, d\theta}{dz} = \dfrac{d_0}{\rho} = d_0 K$, $K = \dfrac{1}{\rho}$ = curvature of beam

⇒ strain is proportional to radius of curvature and distance from neutral axis

we can calculate strain $\epsilon = d_0 \dfrac{\partial^2 u}{\partial z^2}$ from the analytical form for $u(z)$ for a doubly clamped beam

$$\epsilon_n(z) = 0.63\,\frac{\beta_n^2 x_0}{L^2}\,(t/2 - d)\Big[\cos\Big(\frac{\beta_n z}{L}\Big) + \cosh\Big(\frac{\beta_n z}{L}\Big)$$
$$- c_n\big(\sin(\beta_n z/L) + \sinh(\beta_n z/L)\big)\Big]$$

here $\beta_n = 4.73$
d = depth
$d_0 = (t/2 - d_0)$

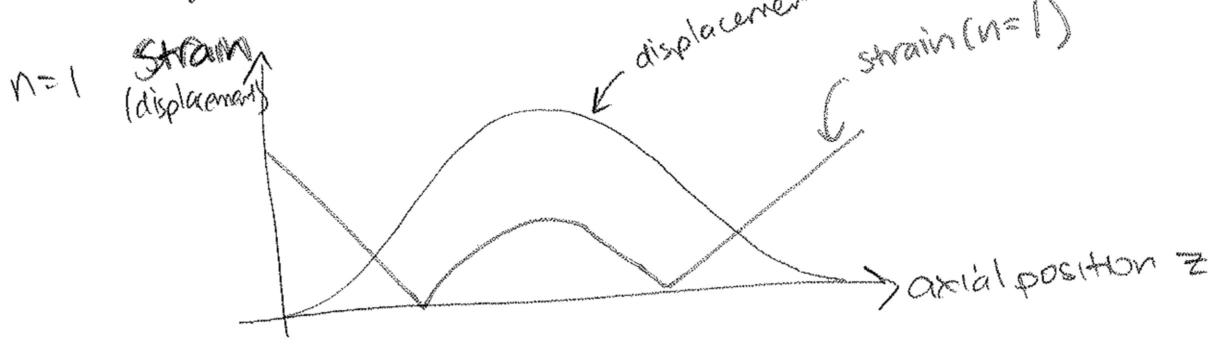
note: linear dependence on X_0. $X_0 = \sqrt{\frac{\hbar}{2 m \omega_0}} \sim \left(\frac{1}{t w L (t/L^2)}\right)^{1/2} \sim \left(L / t^2 w\right)^{1/2}$

strain maximized @ base

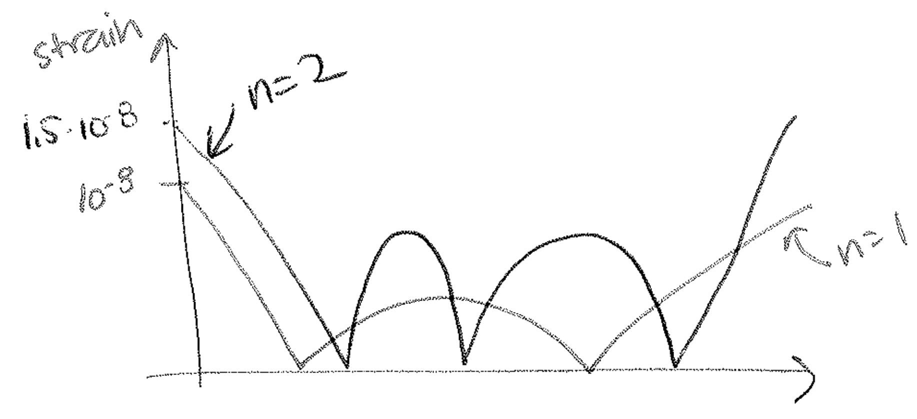
$\epsilon_n(0, d_0 = 0) \approx \frac{x_0 t}{L^2} \sim \frac{1}{L^{3/2} w^{1/2}}$

⇓

want short & narrow cantilevers and NV right at surface
↓
watch out for clamping loss!



beam dimensions
(2um, 100nm, 50nm)
NV @ surface



note! higher order modes have higher zpm strain!

just for fun, lets also look at compressional modes. these have the advantage that NV does not have to be right at surface to maximize strain.

modeshape of a compressional mode: $u(z) = x_0 \sin(n\pi z/L)$

this is a longitudinal mode

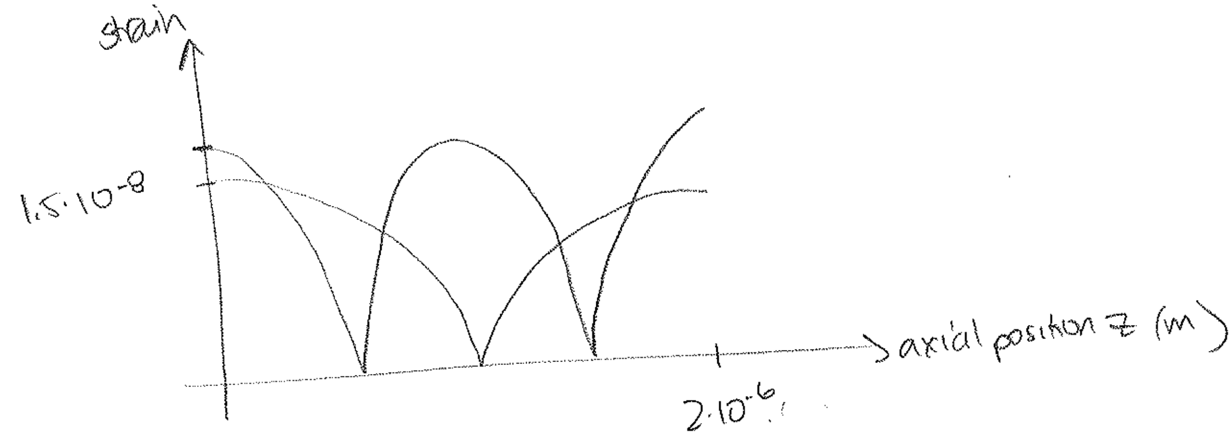
$$\epsilon(z) = du/dz = \frac{n\pi x_0}{L} \cos(n\pi z/L)$$

$$x_0 = \sqrt{\frac{h}{2n\pi^2 \sqrt{E\rho}\, wt}}$$

(obtained by normalization thru integration of strain energy density)

$$\Rightarrow \epsilon_{max} \sim x_0/L \sim \frac{1}{L\sqrt{wt}}$$

$\Rightarrow$ again, want short, narrow, thin

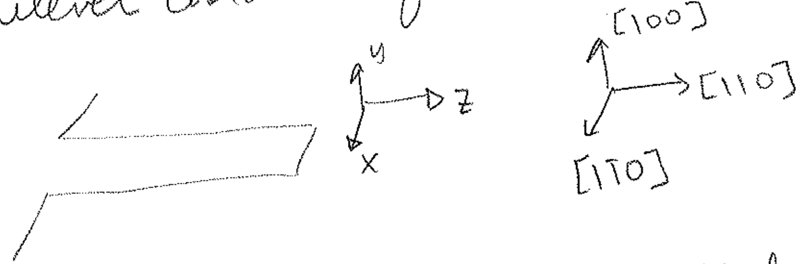


comparable strain.

but Q?

and compressional mode is an order of magnitude higher frequency our simple equations for strain were for the strain along one-axis (the cantilever axis), now lets calculate the whole strain tensor in the NV's coordinate frame (not the cantilever coordinate frame):

cantilever basis



as cantilever bends, stress is all along z

strain tensor in cantilever basis
$$\Xi = \begin{pmatrix} -\nu\epsilon(z) & 0 & 0 \\ 0 & -\nu\epsilon(z) & 0 \\ 0 & 0 & \epsilon(z) \end{pmatrix}$$

$\nu = 0.11$ for diamond

note: need to consider Poisson ratio

we need to transform this strain tensor into a strain tensor in each of the 4 possible NV orientations (where z will lie along NV axis and x will lie along a projection of a carbon bond in the plane $\perp$ to the NV axis

will transform by $\Xi_{NV} = Q\,\Xi\,Q^T$ Q are rotation matrices

where e.g. $Q_{111} = R_z(180°)R_y(54.5°)R_x(90°)$ (could also use Euler angles)

↳ rotation matrix that takes you from 110 → 111

$R_i(\theta)$: rotation by θ along axis i

note: (111) refer to Miller indices

$$\Xi_{111,\bar{1}\bar{1}1} = \begin{bmatrix} (.33-.67\nu)\epsilon(z) & 0 & -.47(1+\nu)\epsilon(z) \\ 0 & -\nu\epsilon(z) & 0 \\ -0.47(1+\nu)\epsilon(z) & 0 & (.67-.33\nu)\epsilon(z) \end{bmatrix}$$

$$\Xi_{\bar{1}1\bar{1},1\bar{1}\bar{1}} = \begin{bmatrix} \epsilon(z) & 0 & 0 \\ 0 & -\nu\epsilon(z) & 0 \\ 0 & 0 & -\nu\epsilon(z) \end{bmatrix}$$

these matrices highlight the fact that different NVs will experience different effects under the same motion of the cantilever

now that we have all the terms in the strain tensor, we want to see how these will affect the NV's energy levels

strain can be expressed in terms of matrices that transform according to the irreducible representations of the point group C_{3v}.

$$\mathcal{H}_{strain} = V'_{A_1}(\epsilon_{xx}+\epsilon_{yy}) + V_{A_1}\epsilon_{zz} \quad \Rightarrow \text{transform as } A_1$$

$$+ V_x(\epsilon_{yy}-\epsilon_{xx}) + V'_x(\epsilon_{xz}+\epsilon_{zx}) \quad \Rightarrow \text{transform as } E_1$$

$$+ V_y(\epsilon_{xy}+\epsilon_{yx}) + V'_y(\epsilon_{yz}+\epsilon_{zy}) \quad \Rightarrow \text{transform as } E_2$$

can strike an equivalence b/w strain and electric field

$$d_{\parallel} E_z = \lambda_1\epsilon_{zz} + \lambda_2(\epsilon_{xx}+\epsilon_{yy})$$

$$d_x E_x = \lambda_3(\epsilon_{yy}-\epsilon_{xx}) + \lambda_4(\epsilon_{xz}+\epsilon_{zx})$$

$$d_y E_y = \lambda_3(\epsilon_{xy}+\epsilon_{yx}) + \lambda_4(\epsilon_{yz}+\epsilon_{zy})$$

in the groundstate (in 0,1,-1 basis)

$$\left.\begin{aligned}
\mathcal{H}_{strain} = &\Big(\overbrace{\lambda_1 \epsilon_{zz} + \lambda_2(\epsilon_{xx}+\epsilon_{yy})}^{=V_{A1},\ \text{shifts } \pm 1 \text{ up}}\Big) S_z^2 \\
&- \Big(\overbrace{\lambda_3(\epsilon_{yy}-\epsilon_{xx}) + \lambda_4(\epsilon_{xz}+\epsilon_{zx})}^{=V_{E_1}}\Big)(S_x^2 - S_y^2) \\
&- \lambda_3\Big(\overbrace{(\epsilon_{xy}+\epsilon_{yx}) + \lambda_4(\epsilon_{yz}+\epsilon_{zy})}^{=V_{E_2}}\Big)(S_x S_y + S_y S_x)
\end{aligned}\right\}\ \text{mixes } +1 \text{ and } -1$$

in excited state (in E_x, E_y basis)

$$\begin{aligned}
\mathcal{H}_{strain} = &\Big[\overbrace{\lambda_1 \epsilon_{zz} + \lambda_2(\epsilon_{xx}+\epsilon_{yy})}^{\text{shifts } E_x + E_y \text{ together}}\Big]\big(|E_x\rangle\langle E_x| + |E_y\rangle\langle E_y|\big) \\
&+ \Big[\overbrace{\lambda_3(\epsilon_{yy}-\epsilon_{xx}) + \lambda_4(\epsilon_{xz}+\epsilon_{zx})}^{\text{brings } E_x + E_y \text{ closer together}}\Big]\Big[|E_x\rangle\langle E_x| - |E_y\rangle\langle E_y|\Big] \\
&+ \underbrace{\big(\lambda_3(\epsilon_{xy}+\epsilon_{yx}) + \lambda_4(\epsilon_{yz}+\epsilon_{zy})\big)}_{\text{mixes } E_x + E_y}\Big[|E_x\rangle\langle E_y| + |E_y\rangle\langle E_x|\Big]
\end{aligned}$$

going back to a simple E field treatment, knowing we can relate E and strain later, let's examine what happens when we strain couple ~~on NV~~ a cantilever to the groundstate spin of an NV

$$\mathcal{H}_{NV} = DS_z^2 + \gamma B - d_\perp \left[E_y(S_xS_y + S_yS_x) + E_x(S_x^2 - S_y^2)\right]$$

S_x, S_y, S_z: SO(3) rep of Pauli matrices

writing $S_\pm = S_x \pm iS_y$ and doing some algebra

$$\mathcal{H}_{strain} = -d_\perp\left(\frac{E_x}{4i}(2S_+^2 - 2S_-^2) + \frac{E_y}{4}(2S_+^2 + 2S_-^2)\right)$$

define $a = E_y - iE_x$
$a^\dagger = E_y + iE_x$

bosonic creation/annihilation operators for mechanical mode
E_x/E_y: x/y quadratures of E field

note: couples +1 and -1! $\leftarrow$ $$\mathcal{H}_{strain} = \frac{d_\perp E_0}{2}\left(|1\rangle\langle -1| a + |-1\rangle\langle 1| a^\dagger\right)$$

E_0: electric field mag associated w/ ZPM of mech mode

apply a B-field that splits $|1\rangle$ and $|-1\rangle$ by Δ_B and bring ω_m into resonance with Δ_B then we can eliminate $|0\rangle$ from picture and make σ^+ and σ^- the SU(2) spin ½ operators:

$\updownarrow \Delta_B = \omega_m$

$$\boxed{\mathcal{H} = \Delta_B \sigma^z + \omega_m a^\dagger a + g(\sigma^+ a + \sigma^- a^\dagger)}$$

now, $d_\perp$ and $d_\parallel$ (to strain) had not been carefully measured
E field susceptibilities had and some made a naive
assumption that $E_\perp / E_\parallel = \sigma_\perp / \sigma_\parallel$ and all the effects
come from the ES (where some strain now to were done) & hence it can be predicted what
$\sigma_\perp, \sigma_\parallel$ in GS would be

here is an example of a measurement of these parameters ~~over~~ for the NV GS
& a demonstration of mechanical coupling
just like B-field AC magnetic moment

when I get to defining qubit pair, use the board
now, it will be instructive to look @ the energy eigenstates

assume $B \parallel$ NV axis ($B_\perp$ small), then one can solve for the
eigenstates of $\mathcal{H}$ in the presence of a $\perp$ E field

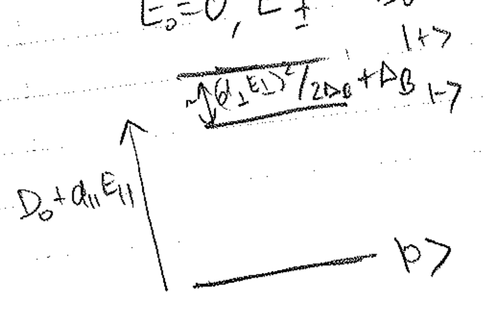
$$|0\rangle$$
$$|+\rangle = \cos\frac{\alpha}{2}\,|1\rangle - e^{-i\phi_s}\sin\frac{\alpha}{2}\,|-1\rangle \qquad \tan\phi_s = E_y/E_x$$
$$|-\rangle = \sin\frac{\alpha}{2}\,|1\rangle + e^{-i\phi_s}\cos\frac{\alpha}{2}\,|-1\rangle$$

$\alpha/2$: Stuckelberg angle, $\tan\alpha = d_\perp E_\perp / \gamma B_z$

(when $E_\perp = 0 \Rightarrow |{-1}\rangle$ and $|1\rangle$ are recovered)

$$E_0 = 0;\; E_\pm = D_0 + d_\parallel E_\parallel \pm \sqrt{(\gamma B_z)^2 + (d_\perp E_\perp)^2} \sim D_0 + d_\parallel E_\parallel \pm \underbrace{\gamma B_z}_{\Delta_B}\left(1 + \frac{1}{2}\left(\frac{d_\perp E_\perp}{\gamma B_z}\right)^2\right)$$
$$= D_0 + d_\parallel E_\parallel \pm \left(\Delta_B + \frac{1}{2}(\ldots\right.$$

$E_\parallel$ shifts levels
$E_\perp$ mixes levels and
splits new levels $|+\rangle$ and $|-\rangle$

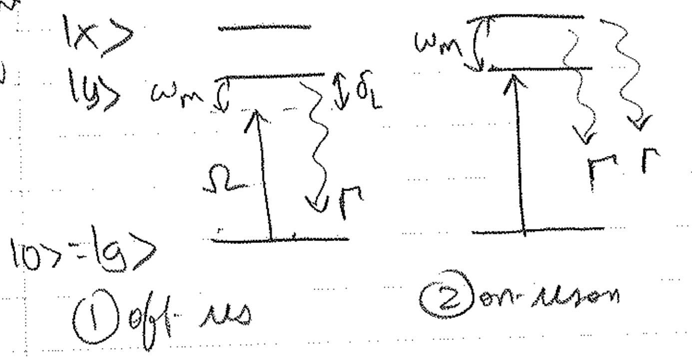


interesting b/c we are adding light into the system in a coherent way which has some int prospects

③ another experiment: coupling mechanical motion to an optical 2LS of the NV. allows for NV-based cooling & also has some int prospects for coupling mechanics & photons thru NV ✓

we will now treat another type of interaction b/w a mechanical degree of freedom & yet another type of "spin" system in the NV. this time the spin system involves the excited state of the NV. one can cool the mechanics via the following 2 types of interactions

I will 1st motivate a theoretical proposal & then discuss relevant details of NV



explain conceptually these 2 schemes

① similar to resolved sideband cooling of atoms a phonon (ω_m) assisted optical transition is made possible when an excitation laser is red-detuned from the $|y\rangle$ excited state

② when a laser excites $|0\rangle \to |y\rangle$ transition there is a finite chance that system absorbs a phonon making a transition to $|x\rangle$ & then relaxing, having effectively absorbed a phonon

the richness of NV ES structure allows such techniques and scheme also relies on fact that strain coupling is much greater ($10^5\times$) in ES than in ground state though Γ_{NV} is much faster, $\sim 10^4 \times$ shorter