Squeezed States of Light

Roman Schnabel

Albert-Einstein-Institut (AEI)
Institut für Gravitationsphysik
Leibniz Universität Hannover
Photo-Electric Current

A (modulation) signal?

Photons per time interval

Time intervals
Photo-Electric Current

No signal, but photon shot noise?

![Diagram showing photon counts per time interval]

Photons per time interval

Time intervals
Photon Counting Statistics

Coherent state $|\alpha\rangle = |100\rangle$

$\Rightarrow \langle \hat{N} \rangle = \bar{N} = |\alpha|^2 = 10000$

Relative shot-noise $\sim \frac{1}{\sqrt{\bar{N}}} \sim \frac{1}{\sqrt{P_{Laser}}}$

![Graph showing probability distribution for photon number $N$.](image)
“Squeezed” Counting Statistics

Photon number $N$

Probability [rel. units]

Squeezing factor: 3 dB
Squeezing factor: 10 dB

Noise squeezing
Photo-Electric Current – Squeezed

Broadband squeezing

![Graph showing time intervals and photons per time interval with broadband squeezing](image)
Squeezing of Interferometer Shot-Noise

C. M. Caves (1981): Reduction of quantum noise with squeezed light

[Caves, Phys. Rev. D 23, 1693 (1981)]
Wigner Function of Squeezed Light

Phase-space quasi-probability distribution of a squeezed vacuum state, $\Omega = 5 \text{ MHz}$

$\frac{\Delta^2 X_1(\Omega, \Delta \Omega)}{\Delta^2 X_1^{\text{vac}}(\Omega, \Delta \Omega)} = 0.07 \equiv -11.6 \text{ dB}$

[Mehmet et al., PRA 81, 013814 (2010)]
Generation of Squeezed Light (PDC)

\(\chi^2\)-nonlinear crystal: MgO:LiNbO_3

Standing wave cavity

Pump field input (cw, 532nm)

Squeezed field output (cw, 1064nm) by parametric down-conversion (PDC)
Detection of Squeezed States

Photograph of the optical resonator components for squeezing generation via **parametric down conversion**. Crystal: PPKTP or MgO:LiNbO₃

**Graphical Description:**
- Resonator for laser frequency $f_0$
- PZT
- AR
- Squeezed light composed of photon pairs, $f_0$
- Pump-light, $2f_0$
- Bright field, $f_0$ (LO)
- Balanced homodyne detector
- $f_{Fou}$
- $\Delta f = 1\text{Hz}$
Generation of Squeezed Light

\[ P(E) = c_0 \left( \chi^{(1)} E + \chi^{(2)} E^2 \right) \]

\[ E^{in} = E_{vac,f}^{in} + E_{2f}^{in} \]

Squeezed States (Overview)

\[ |\alpha, \xi\rangle , \quad \xi = r \cdot e^{i\theta} \]  
Squeezing parameter and angle

\[ |0, \xi\rangle = \hat{S}(\xi)|0\rangle = \exp \left[ \frac{1}{2} \left( \xi^* \hat{a}^2 - \xi \hat{a}^+ \right) \right]|0\rangle \]  
Squeezing operator producing squeezed vacuum

\[ \langle \alpha, \xi|\Delta^2 \hat{X}_1|\alpha, \xi\rangle = \frac{1}{4} e^{-2r} \]  \[ \langle \alpha, \xi|\Delta^2 \hat{X}_2|\alpha, \xi\rangle = \frac{1}{4} e^{2r} \]  
Quadrature variances

\[ \hat{S}^+(\xi)\hat{a} \hat{S}(\xi) = \hat{a} \cosh r - \hat{a}^+ e^{i\theta} \sinh r \]  
\[ \hat{S}^+(\xi)\hat{a}^+ \hat{S}(\xi) = \hat{a}^+ \cosh r - \hat{a} e^{-i\theta} \sinh r \]  
Eigenwert equation

\[ \hat{\rho}_{0,\xi} = |0, \xi\rangle \langle 0, \xi| , \quad \langle m | \hat{\rho}_{0,\xi} | n \rangle \equiv \rho_{0,\xi,m,n} = \cdots \]  
Density matrix element in Fock basis

\[ \langle 2m + 1 | \hat{\rho}_{0,\xi} | 2m + 1 \rangle = 0 \]

\[ \langle 2m | \hat{\rho}_{0,\xi} | 2m \rangle = \frac{(2m)!}{2^{2m} (m!)^2} \frac{(\tanh r)^{2m}}{\cosh r} \]
“Squeezing” Laboratories

First squeezed light: [Slusher et al., PRL 55, 2409 (1985)]

Research labs with squeezed light (not complete):
- Kimble (CalTech): \textit{teleportation}: [Furuzawa et al., SCIENCE 282, 706 (1998)]
- Grangier (Orsay); \textit{kitten}: [Ourjoumtsev et al., SCIENCE, 312, 83 (2006)]
- Schiller and Mlynek (Konstanz): \textit{tomography}: [Nature 387, 471 (1997)]
- Barchor and Lam (Canberra): \textit{6dB at 1064nm} [J. Opt. B 1, 469 (1999)]
- Leuchs (Erlangen); \textit{~7 dB pulsed} [Opt. Lett. 33, 116 (2008)]
- Polzik (Copenhagen), [Neergaard-Nielsen et al., PRL 97, 083604 (2006)]
- Furusawa (Tokyo); \textit{9 dB}: [Takeno et al., Opt. Express 15, 4321 (2007)]
- Fabre (Paris); Zhang, Peng (Shanxi); Andersen (Copenhagen); Mavalvala (MIT)
- Nussenzveig (Sao Paulo); Pfister (Virginia); ...
Squeezing Issues for GW Detection

Squeezing at frequencies in the GW detection band (10 Hz to 10 kHz)
- First Audioband squeezing [McKenzie et al., PRL 93, 161105 (2004)]
- New control scheme [Vahlbruch, RS et al., PRL 97, 011101 (2006)]
- 6 dB over complete band [Vahlbruch, RS et al., NJP 9, 371 (2007)]

Compatibility with GW detector techniques
- Power-recycling [McKenzie et al., PRL 88, 231102 (2002).]
- Signal-recycling [Vahlbruch, RS et al., PRL 95, 211102 (2005)]
- Suspended interferometer [Goda et al., Nat. Phys. 4, 472 (2008).]

Strong continuous wave squeezing (>10 dB) at 1064nm
- [Vahlbruch, RS et al., PRL 100, 033602 (2008)]
- [M. Mehmet, RS et al., PRA 81, 013814 (2010)]

2007: Squeezing Down to 1 Hz

(d) Vacuum noise level, $P_{\text{LO}} = 464 \, \mu\text{W}$
(e) Squeezed noise, $P_{\text{LO}} = 464 \, \mu\text{W}$
(f) Electronic dark noise


First audio-band squeezing:
[McKenzie et al., PRL 93, 161105 (2004)]
2008: Squeezing Generation >10dB

[H. Vahlbruch, RS et al., PRL 100, 033602 (2008)]
Design of the “GEO600 Squeezer”

[Henning Vahlbruch, PhD thesis, Hannover, 2008]
   - Gravitational Wave International Committee (GWIC) Thesis Prize 2008
   - S-AMOP DPG Thesis Prize 2010
The GEO600 Squeezed Light Laser

Alexander Khalaidovski and Henning Vahlbruch, 2009/2010
Real-time control system: Nico Lastzka and Christian Gräf
The GEO600 Squeezed Light Laser


Transport to the GEO600 GW Detector
The Implementation

April 2010 – September 2010:

• Implementation of an output mode-cleaning cavity
• Phase lock of the Squeezed-Light Laser to the GEO600 main laser
• Coupling into GEO600 via a low-loss Faraday rotator (custom design with <1% loss)
• Matching of the squeezed mode to the GEO600 mode
• Replacement of photo diode (custom design with 3mm ⊗ and >99% quantum efficiency)

[The “GEO600 Team” led by Hartmut Grote and the “Squeezer Team” led by Henning Vahlbruch]
GEO600: Noise without Squeezed Light

Observatory noise without squeezed light
GEO600: Squeezing Enhanced

[The LSC, Nat. Phys. 7, 962 (2011)]

Observatory noise without squeezed light

Observatory noise with squeezed light (and loss)
A gravitational wave observatory operating beyond the quantum shot-noise limit

The LIGO Scientific Collaboration

This is not just a proof of principle! GEO600 regularly uses squeezed light during its observational runs.
Squeezed States of Light

\[ P \rightarrow D \]

\[ P_{\text{light}}^{(2)} \sim E_{\text{light}}^2(t) \sim I_{\text{light}}(t) \sim V(t) \]

In laser interferometry, we are usually interested in modulations of the photo-electric voltage \( V(t) \).

Gravitational wave detectors (gravimeters).

Modulations: \( 10 \text{ kHz} - 20 \text{ kHz} \).
\[ \frac{\Omega}{2\pi} \]

Amplitude-modulable (AM)

\[ \frac{\omega_0}{2\pi} = \frac{c}{\lambda} \]

\[ \lambda = 1064 \text{ nm} \]
The complex amplitude $a$

(defined to describe the amplitude across the phase of field)

$\omega$: local oscillator

$\omega_0$: DC, CO

Upper sideband (+$\omega_0$)

$\omega_0$: DC stands still in frame rotating with $\omega_0 = \frac{c \cdot 2\pi}{\lambda}$

$\lambda$: DC, CO
\[ AM: \quad \frac{x_l + x_u^*}{2} \equiv x_1 \cos (\theta_0 + \phi_1) \]

\[ PM: \quad -i \frac{x_l^* - x_u}{2} \equiv x_2 \cos (\theta_0 + \phi_2) \]

\[ \Rightarrow \quad \alpha = \alpha_{dc} + x_1 \cos (\theta_0 + \phi_1) + i x_2 \cos (\theta_0 + \phi_2) \]

\[ \uparrow \quad \text{amplitude of } AM \quad \uparrow \quad \text{phase of } AM \quad \uparrow \quad \text{amplitude of phase} \]

\[ \text{given}\]
Photo voltage (band around $\Omega_0$)

$V_{\Omega_0}(\tau) \sim \alpha^* \alpha$

\[ = \alpha^2_{\text{DC}} + 2 \alpha_{\text{DC}}^2 \cos(\theta^0 + \phi_1) + \chi^2 \cos^2(\theta^4 + \phi_1) \]

This part contains information about all in a linear way.

How to extract this $\uparrow$ in an experiment?
"Mixing" the photo voltage with an electronic local oscillator (LO): \( A \cdot \cos(\phi_0 + \Phi) \)

Setting: \( \Phi = 0, \phi_0 = 0 \)

\[ V_{2o}(+ \theta) = A \cdot \cos(\phi_0 + \Phi) \]

\[ \sim \lambda_{DC} \cdot A \cdot \cos(\phi_0 + \Phi) + 2 \lambda_{DC} \cdot A \cdot \cos^2(\phi_0 + \Phi) \cdot A \]

\[ + \text{terms} \left\{ \cos^3(\phi_0 + \Phi), \cos(\phi_0 + \Phi) \right\} \]

\[ = \sim \sim \sim 2 \lambda_{DC} \cdot A \left( \frac{1}{2} + \frac{1}{2} \cos(2\phi_0 + \Phi) \right) \]

\[ + \text{terms} \left\{ \sim \sim \sim \right\} \]
Low-Pass Filtering

$\Delta \Omega < \Omega_0$ is the "resolution bandwidth" of our measurement.

\[
V_{\text{filtered}}(t) \sim A \cdot x_{dc} \cdot X_{\Omega}(\omega_0, \Delta \Omega, t)
\]

$X_{\Omega}(\omega_0, \Delta \Omega, t)$ is the modulation depth of the AM and also called "amplitude modulation amplitude."
\[ D \xrightarrow{V(t)} \mathbb{C} \xrightarrow{V_{n_0}} H \sim X_n(p, \Delta, \rho,t) \]

\[ A \cdot \cos(\varphi + \bar{\varphi}) \]
Balanced homodyne detector

\[ V_1(\tau) = \alpha^\dagger_1 \cdot \alpha_1 \]
\[ = \frac{1}{\sqrt{2}} \left( \alpha^* + e^{-i\theta} \alpha_{10} \right) \cdot \frac{1}{\sqrt{2}} \left( \alpha + e^{i\theta} \alpha_{10} \right) \]

\[ V_2(\tau) = \alpha^\dagger_2 \cdot \alpha_2 \]
\[ = \frac{1}{\sqrt{\eta}} \left( \alpha^* - e^{-i\theta} \alpha_{10} \right) \cdot \frac{1}{\sqrt{\eta}} \left( \alpha - e^{i\theta} \alpha_{10} \right) \]
\[ X(0) \xrightarrow{DC} X(\alpha), \quad 0 \leq \alpha \leq \theta \]

\[ V_x(t) \sim A \cdot L \cdot \left[ X_{1}\left(0, \alpha \right) \cos \theta \right. \]
\[ \left. + X_{2}\left(\rho_0, \alpha t, \theta\right) \sin \theta \right] \]

\[ \equiv X_{\theta}\left(\rho_0, \Delta \iota, \theta\right) \]
For $\theta = \frac{\pi}{2}$ you measure

$X_2 (R, \Delta R, t)$ being the

"phase grader" applied.

Quantum Optics

There are uncertainties on $X_1$ and $X_2$.

Zero point: $\Delta x_{1,\text{vac}} (R, \Delta R) = \Delta x_{2,\text{vac}} (R, \Delta R) = \frac{1}{4}$

Fluctuation:

$[X_{1, \text{vac}} X_{2, \text{vac}}] = \frac{1}{2}$

Heisenberg uncertainty relation $\Delta x_{1} \Delta x_{2} \geq \frac{1}{4}$
Squeezing, see pour point penetration