Quantum Noise in Gravitational Wave Detectors

Roman Schnabel

Albert-Einstein-Institut (AEI)
Institut für Gravitationsphysik
Leibniz Universität Hannover
Michelson Laser Interferometer

1) Photon counting noise
2) Back-action

- Laser
- Mirror 1
- Mirror 2
- Frequency lock
- Squeezed light laser
- Faraday Rotator
Measurement Noise and Back-Action

Quantum noise in phase quadrature

- Shot noise
- Radiation pressure noise
- Standard quantum limit (SQL)
- 20dB squeezed
  (Alternatively 100x laser power)

[Caves 1981]
The SQL – an ultimate Limit?

PHYSICAL REVIEW LETTERS

Volume 54  10 June 1985  Number 23

Defense of the Standard Quantum Limit for Free-Mass Position

Carlton M. Caves

Theoretical Astrophysics, California Institute of Technology, Pasadena, California 91125
(Received 6 April 1984)

Measurements of the position $x$ of a free mass $m$ are thought to be governed by the standard quantum limit (SQL): In two successive measurements of $x$ spaced a time $\tau$ apart, the result of the second measurement cannot be predicted with uncertainty smaller than $(\hbar \tau / m)^{1/2}$. Yuen has suggested that there might be ways to beat the SQL. Here I give an improved formulation of the SQL, and I argue for, but do not prove, its validity.

[C. M. Caves, Phys. Rev. Lett. 54, 2465 (1985)]
Squeezing SN and RPN

[Jaekel, Reynaud 1990]

Standard Quantum Limit (SQL)

Squeezed Light Input (8dB)

QND-Regime

Shot noise

Radiation pressure
Quantum Limits in Interferometric Measurements.

M. T. Jaeckel(*) and S. Reynaud(**)

(*) Laboratoire de Physique Théorique de l’Ecole Normale Supérieure(§)
24 rue Lhomond, F-75231 Paris Cedex 05

(**) Laboratoire de Spectroscopie Hertzienne($$), Université Pierre et Marie Curie
4 place Jussieu, F-75252 Paris Cedex 05

“Photon counting noise and radiation pressure noise in a GW detector can both be squeezed!”

First doubts on the free-mass SQL as the ultimate limit:


Squeezing SN and RPN: Filter cavities


Laser

Faraday Rotator

Filter cavities

Squeezed light laser

Squeezing and Full Evasion of RPN

„Squeezed Variational Output“


Optical quality of filters even more demanding!
Squeezing and Full Evasion of RPN

- Back Action Evasion
- Squeezed Light Input
- QND-Regime

Linear noise spectral density [1/√Hz]

Frequency [Hz]

„Squeezed Variational Output“

(SQL)
Ponderomotive Squeezing

- Vacuum uncertainty
- Back-action cancellation
- Additional phase noise due to radiation pressure (amplitude coupling here: 1)
Ponderomotive Squeezing

- Back-action cancellation
- Additional phase noise due to radiation pressure (amplitude coupling here: 1)
Ponderomotive Squeezing

- Is of rather limited strength (requires RPN being dominant
- has a frequency dependent angle (and strength)
- good to cancel back-action, less good as a squeezing source

Recent experiments:
- Observation of radiation pressure noise on a membrane at NIST

- Observation of ponderomotive squeezing at Caltech
Quantum noise in GW detectors

1) Photon counting noise (no real mirror displacement)
2) Radiation pressure noise
   (momentum transfer from light to mirror)
(3) Zero-point fluctuation of the mirror
    is not an issue!

"Optomechanics does not need an optical cavity"
Again: We are interested in modifications of the light power!

1) Photon counting noise has a white spectrum!

\[ \text{Shot noise per } \Delta S = 1 \text{ Hz equivalent to a variance of apparent linear displacement } \Delta x \]

in \( \text{m}^2 \) for a simple Michelson interferometer

\[ \Delta x_{\text{SN}} = \frac{t_a c \cdot 2}{4 \pi P_0} = S_{x,\text{SN}} \quad \text{in} \quad \left[ \frac{\text{m}^2}{\text{Hz}} \right] \]

Displacement normalized (single-sided) power spectral density.
2) Radiation pressure noise

\[ \vec{T} \propto \frac{2}{c} P_0 \]  
\[ \text{retro-reflection} \]
\[ \text{lift force reflector} \]

Quantum uncertainty

\[ \Delta^2 P_0 = 2 \hbar \omega P_0 \equiv S_{P,SN} \propto \frac{U^3}{\hbar T} \]

\[ S_{T,SN} = \left( \frac{2}{c} \right)^2 S_{P,SN} = \frac{8 \hbar \omega P_0}{c^2} \propto \frac{N^2}{\hbar T} \]

Independent of Fourier frequency \( \omega \)
\( \equiv \) white spectrum
The effect of the back-action force on mirror displacement (rpm)

- effect is stronger the smaller the mass
- effect strong on mechanical resonators

\[ S_{x, \text{rpm}} = |H(\nu)|^2 \cdot S_{x, \nu} \]

\[ = \left| \frac{-1}{m(S^2 + i\nu \tau - \omega_{\text{res}}^2)} \right|^2 \cdot \frac{8\pi \omega P_0}{c} \]

"Radiation pressure noise"
Back-action is not Unit

\[
\log_{10} \sqrt{Sx_r \rho_\Pi}
\]
For $\lambda > \lambda_{cr}$,

\[
\sqrt{S_{x, \nu m}} \approx \frac{1}{\omega \sqrt{\sigma}} \sqrt{\frac{8 \pi \omega P_0}{c^2}}
\]

Re calibration: \( \Delta h = \Delta x \frac{1}{c^2} \)

\[
\sqrt{S_{h, \nu m}} \approx \frac{1}{\omega \sqrt{\sigma}} \sqrt{\frac{8 \pi \omega P_0}{c^2}} \quad \Delta H = \sqrt{\frac{1}{\pi \omega^2}}
\]
GUD Quantum Noise

Total quantum noise assuming SN and RPN are uncorrelated.

1σ in the arms uncertainty!
The SQL for a force measurement

Assume uncorrelated SN and RPN

\[ \text{add variances of the two and find the minimum when varying } P_0 \]

\[ \left( \frac{hc^2}{2uP_0} + \frac{8\pi\omega P_0}{c^2 m^2 \Omega^2} \right) \]

\[ \text{derivative} \]

\[ P_0, \text{opt} = \frac{c^2 m^2 \Omega^2}{4\omega} \]

depends on frequency

\[ c = \frac{2.\pi}{\lambda} \] (speed of wave length)
\[
S_{X,\text{SQL}} = \frac{4k}{mN^2}
\]

Spectrum of SQL

\[
\therefore S_{X,\text{SN}} = S_{X,\text{NLL}}
\]

\[
= \frac{1}{2} S_{X,\text{SQL}}
\]
The SQU can be squeezed [Jacak et al., 2002] using squeezed light by a frequency dependent squeezing angle.

Backaction can be evaded completely by using output filter cavities, "variahonal output."

The lattice is linked to **pandemus** t sperm.

See PowerPoint slide **end**