Quantum Noise in Gravitational Wave Detectors

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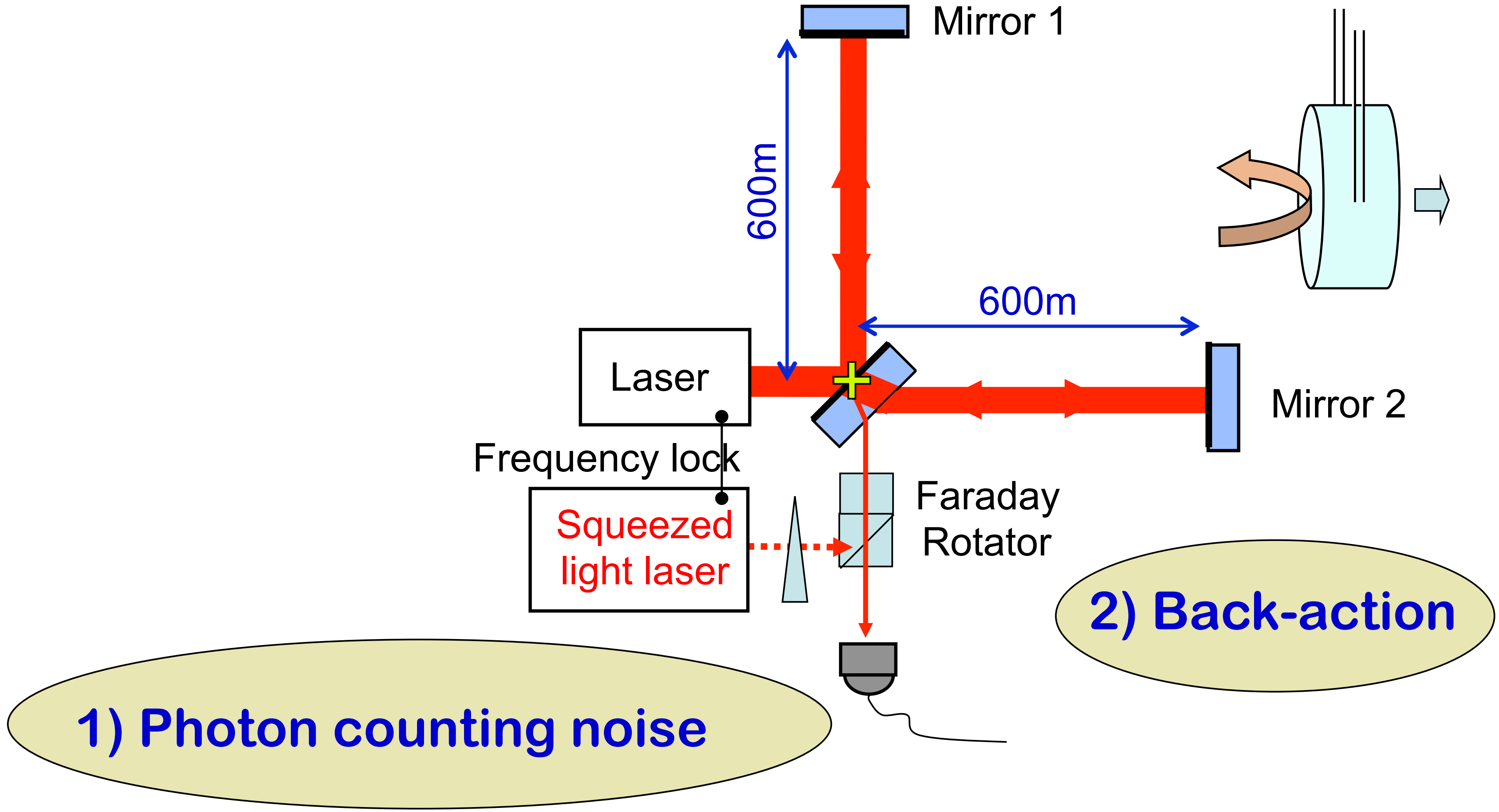
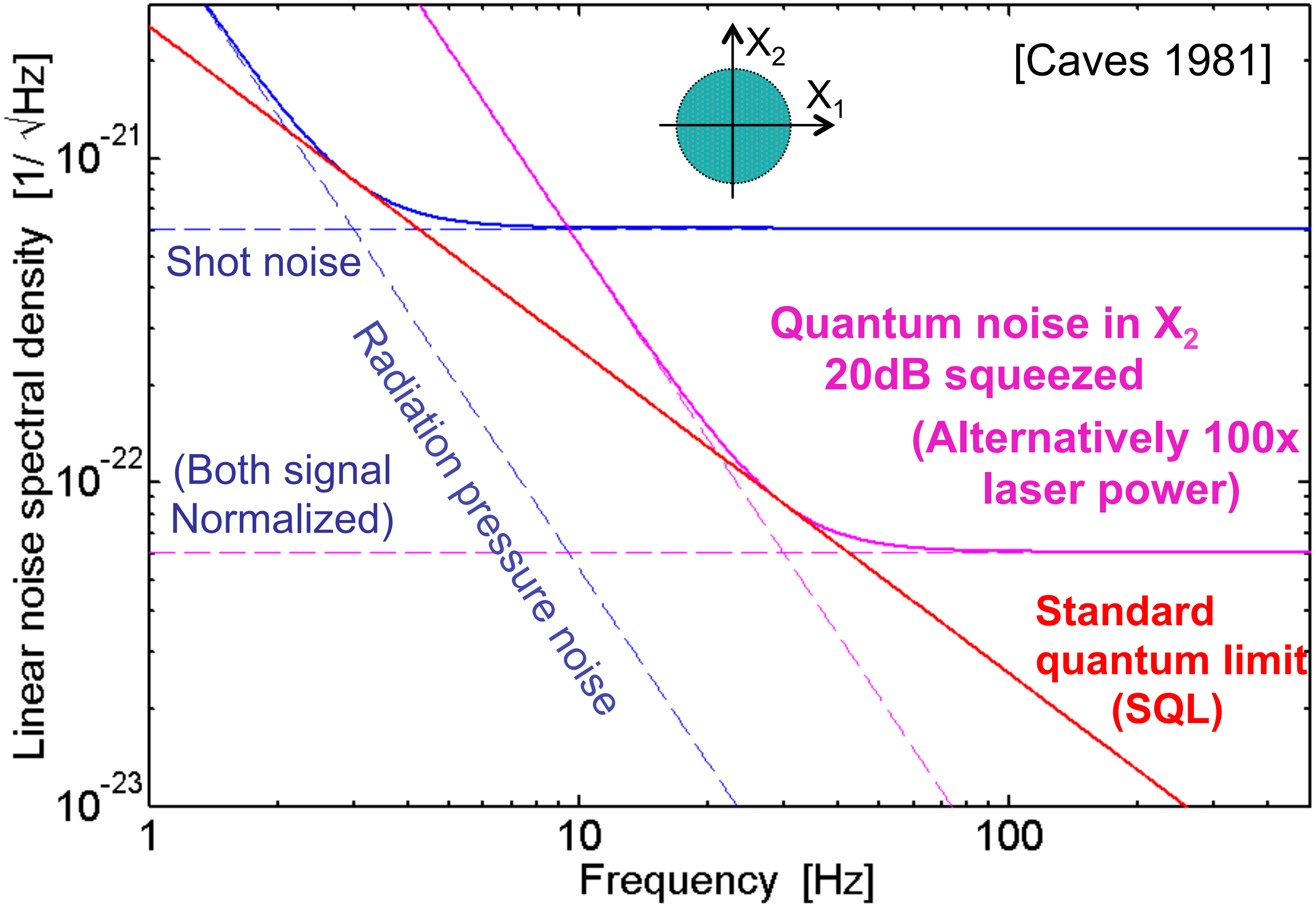
Michelson Laser Interferometer



Measurement Noise and Back-Action



The SQL – an ultimate Limit?

PHYSICAL REVIEW
LETTERS

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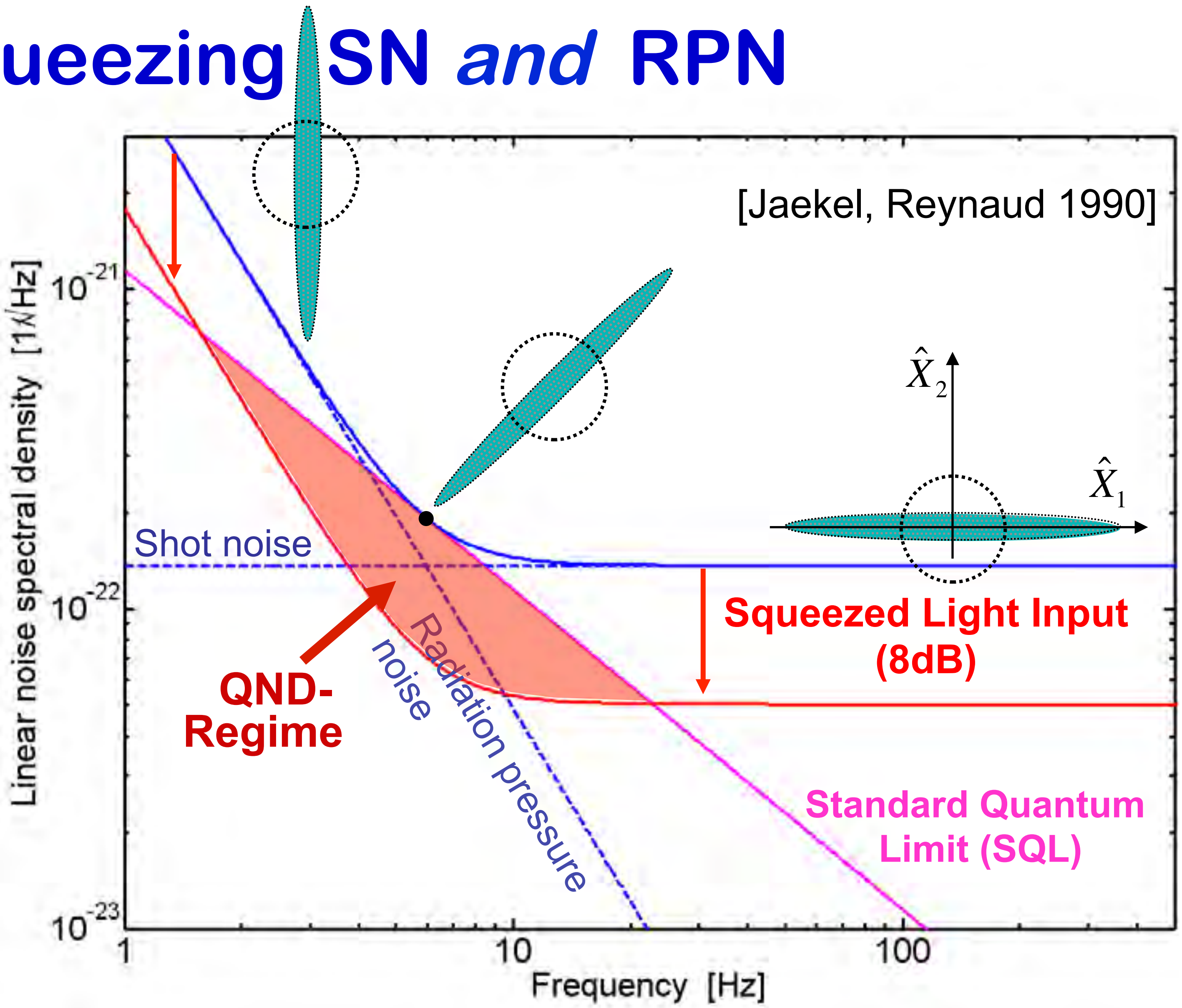
Defense of the Standard Quantum Limit for Free-Mass Position

Carlton M. Caves
Theoretical Astrophysics, California Institute of Technology, Pasadena, California 91125
(Received 6 April 1984)

Measurements of the position x of a free mass m are thought to be governed by the standard quantum limit (SQL): In two successive measurements of x spaced a time τ apart, the result of the second measurement cannot be predicted with uncertainty smaller than $(\hbar\tau/m)^{1/2}$. Yuen has suggested that there might be ways to beat the SQL. Here I give an improved formulation of the SQL, and I argue for, but do not prove, its validity.

[C. M. Caves, *Phys. Rev. Lett.* **54**, 2465 (1985)]

Squeezing SN *and* RPN



EUROPHYSICS LETTERS 15 October 1990

Europhys. Lett., 13 (4), pp. 301-306 (1990)

Quantum Limits in Interferometric Measurements.

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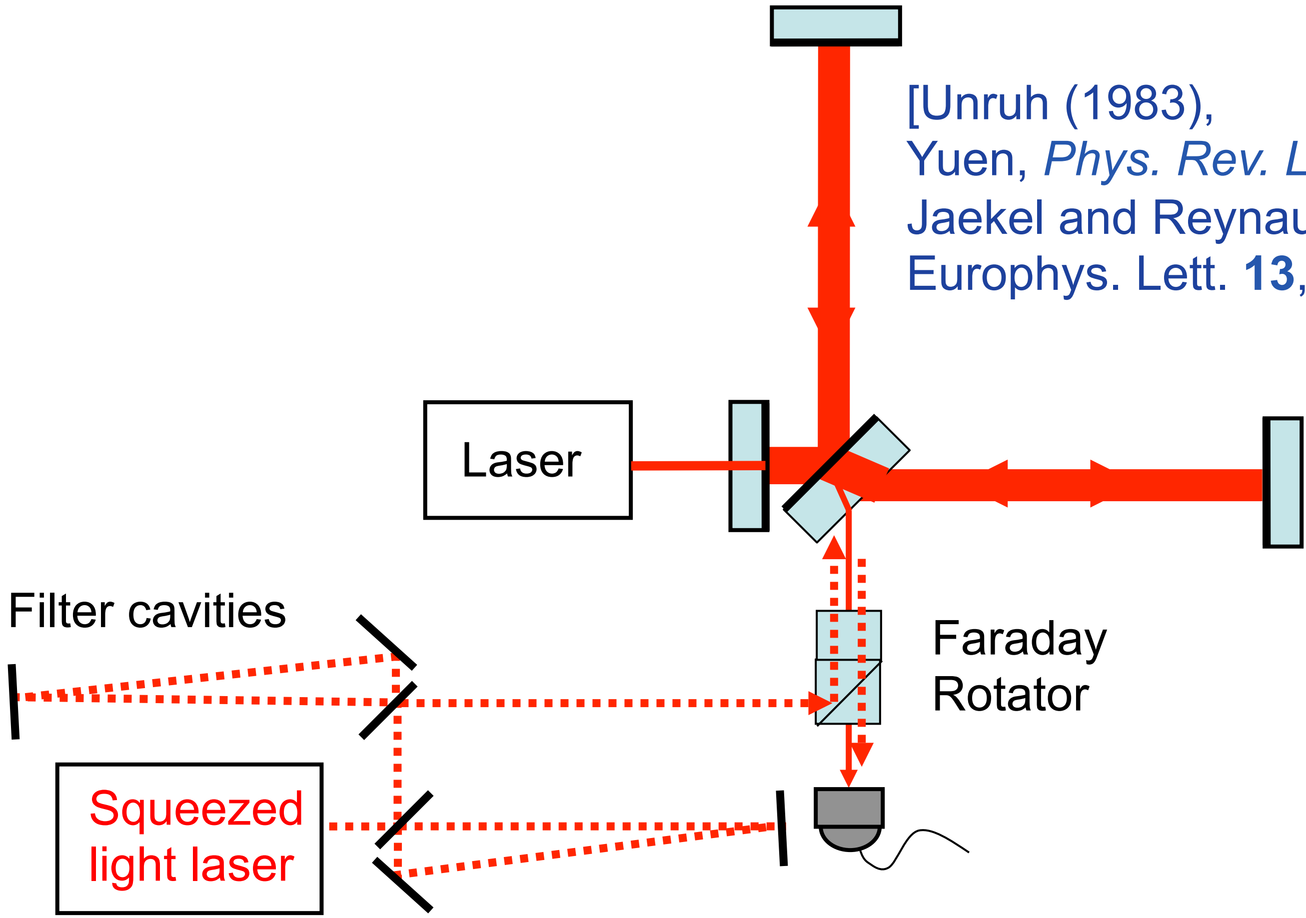
"Photon counting noise and radiation pressure noise in a GW detector can ***both*** be squeezed!"

First doubts on the free-mass SQL as the ultimate limit:

W. G. Unruh, in *Quantum Optics, Experimental Gravitation, and Measurement Theory* 647–660 (Plenum, 1983),

H. P. Yuen, Phys. Rev. Lett. **51,** 719 (1983)

Squeezing SN *and* RPN: Filter cavities

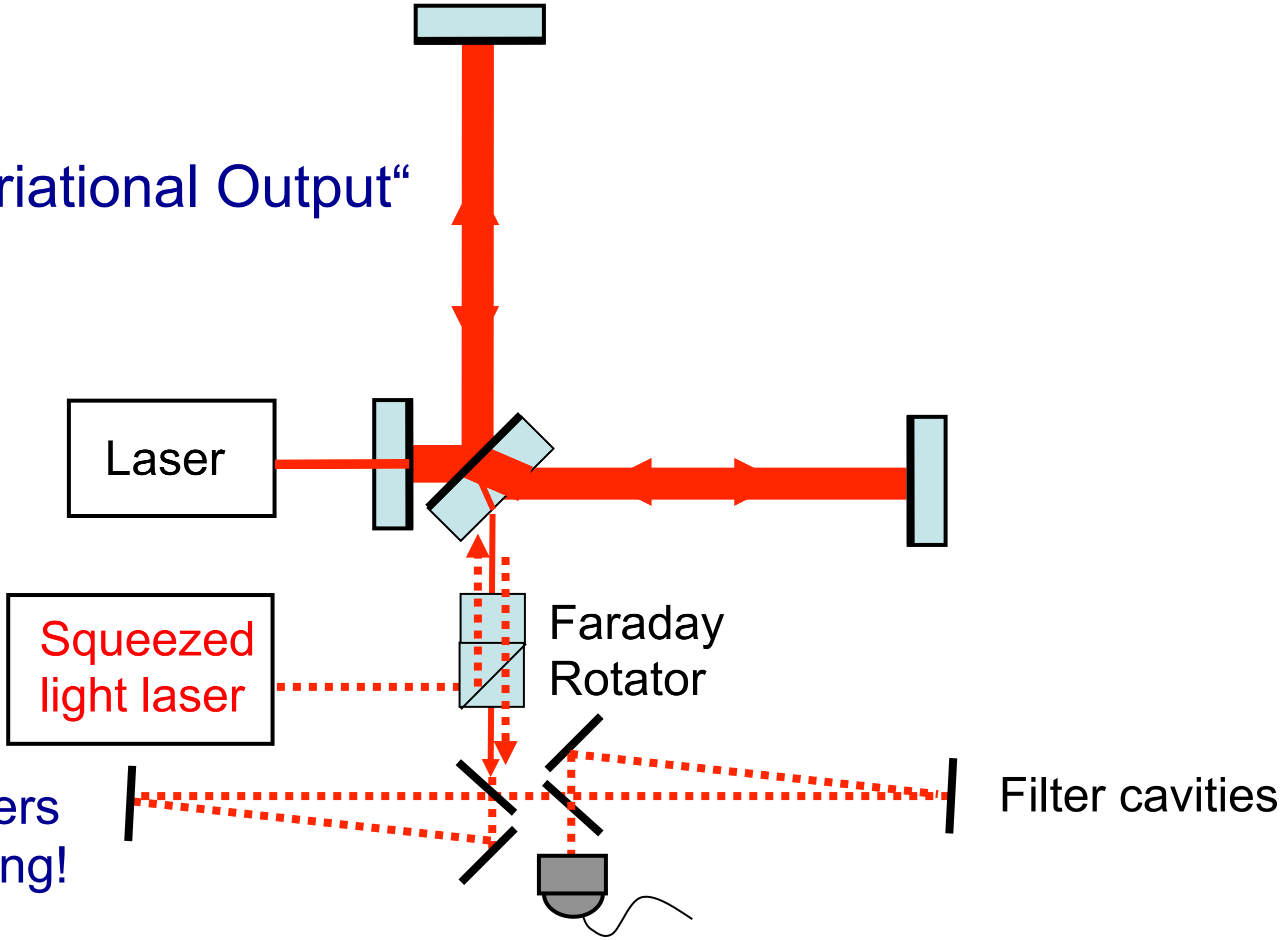


[Unruh (1983),
Yuen, *Phys. Rev. Lett.* **51,** 719 (1983);
Jaekel and Reynaud,
Europhys. Lett. **13**, 301 (1990)]

[Kimble *et al.*, Phys. Rev. D **65**, 022002 (2001)]

Squeezing and Full Evasion of RPN

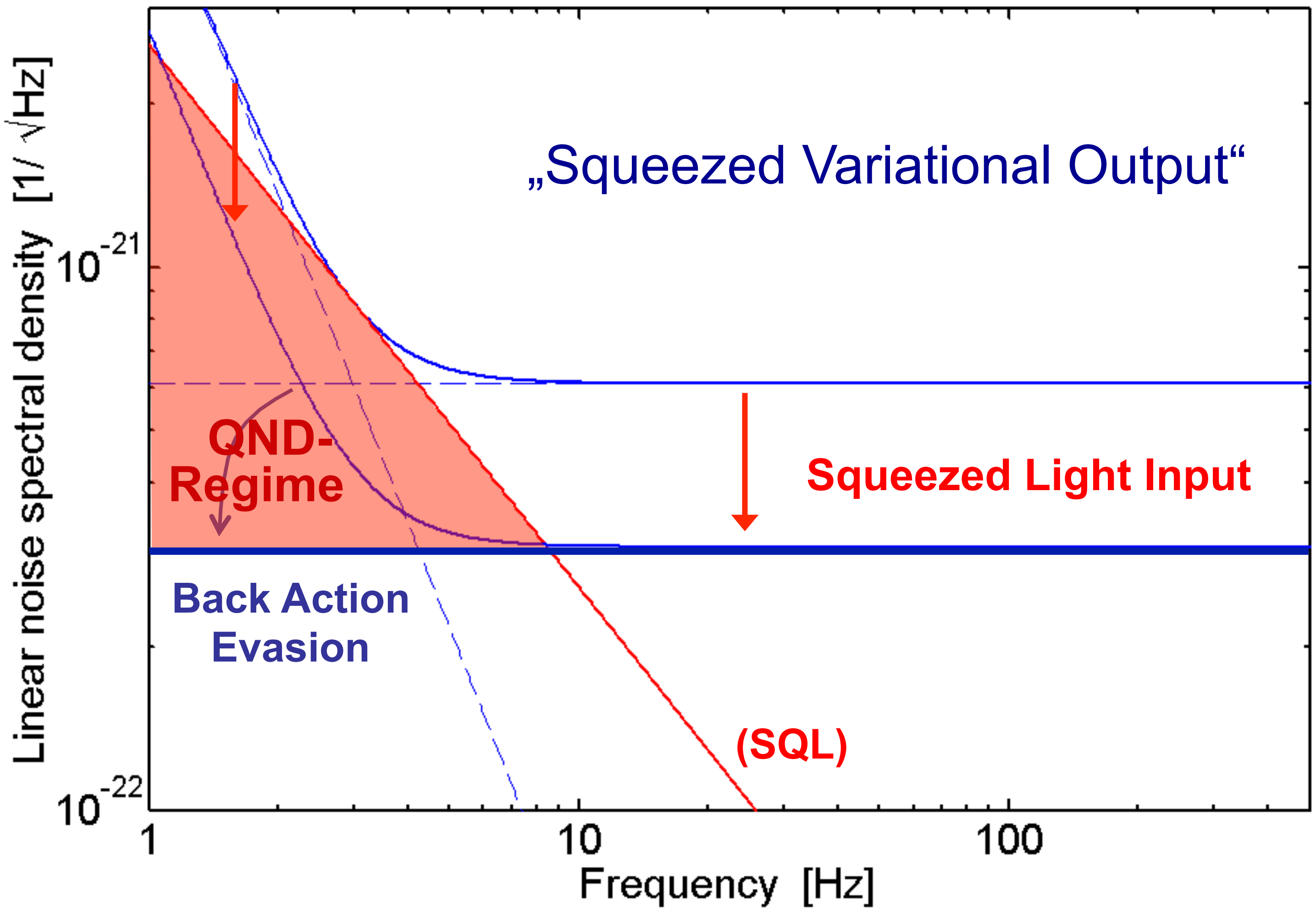
„Squeezed Variational Output"



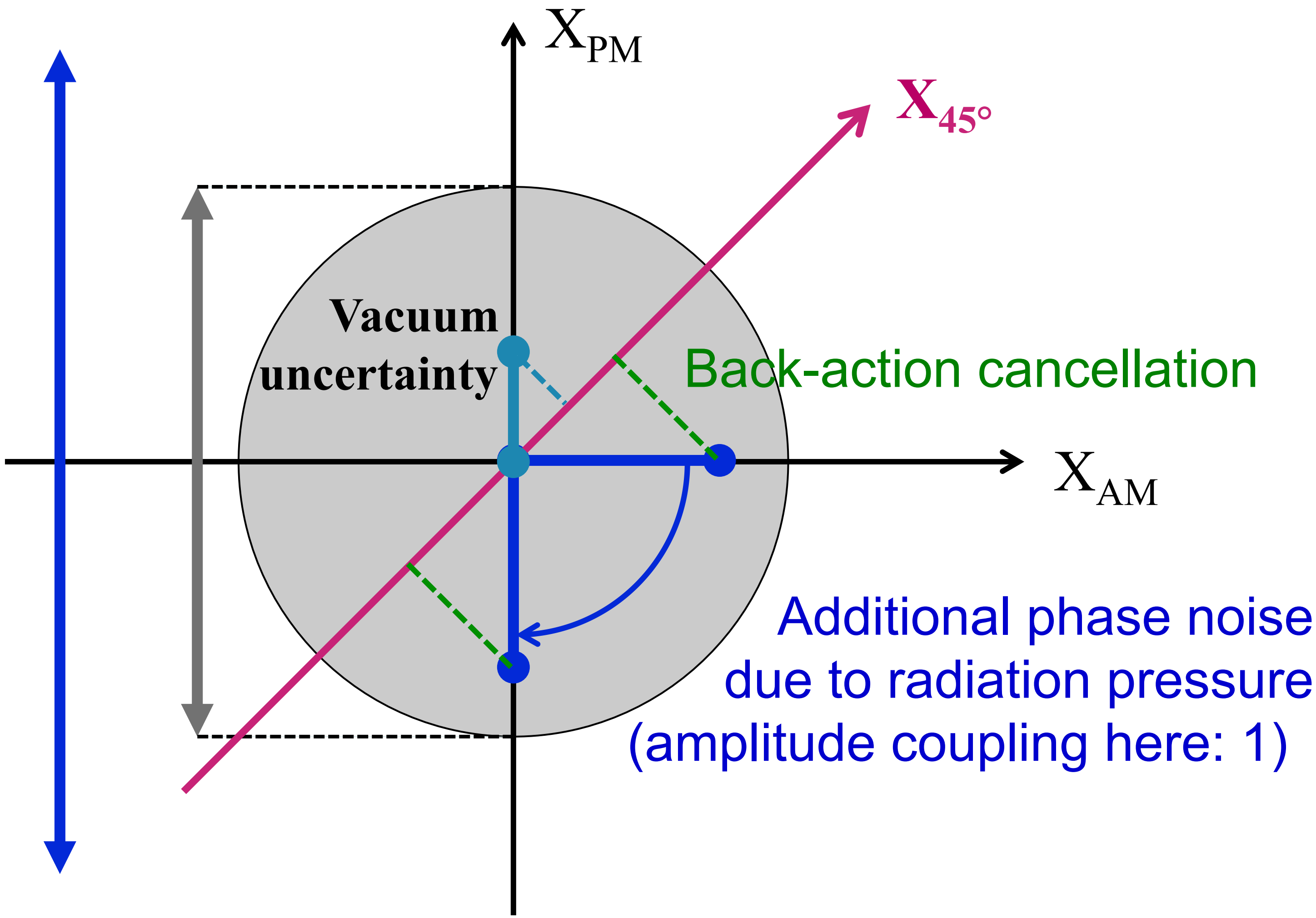
Optical quality of filters even more demanding!

[Kimble *et al.*, Phys. Rev. D **65**, 022002 (2001)]

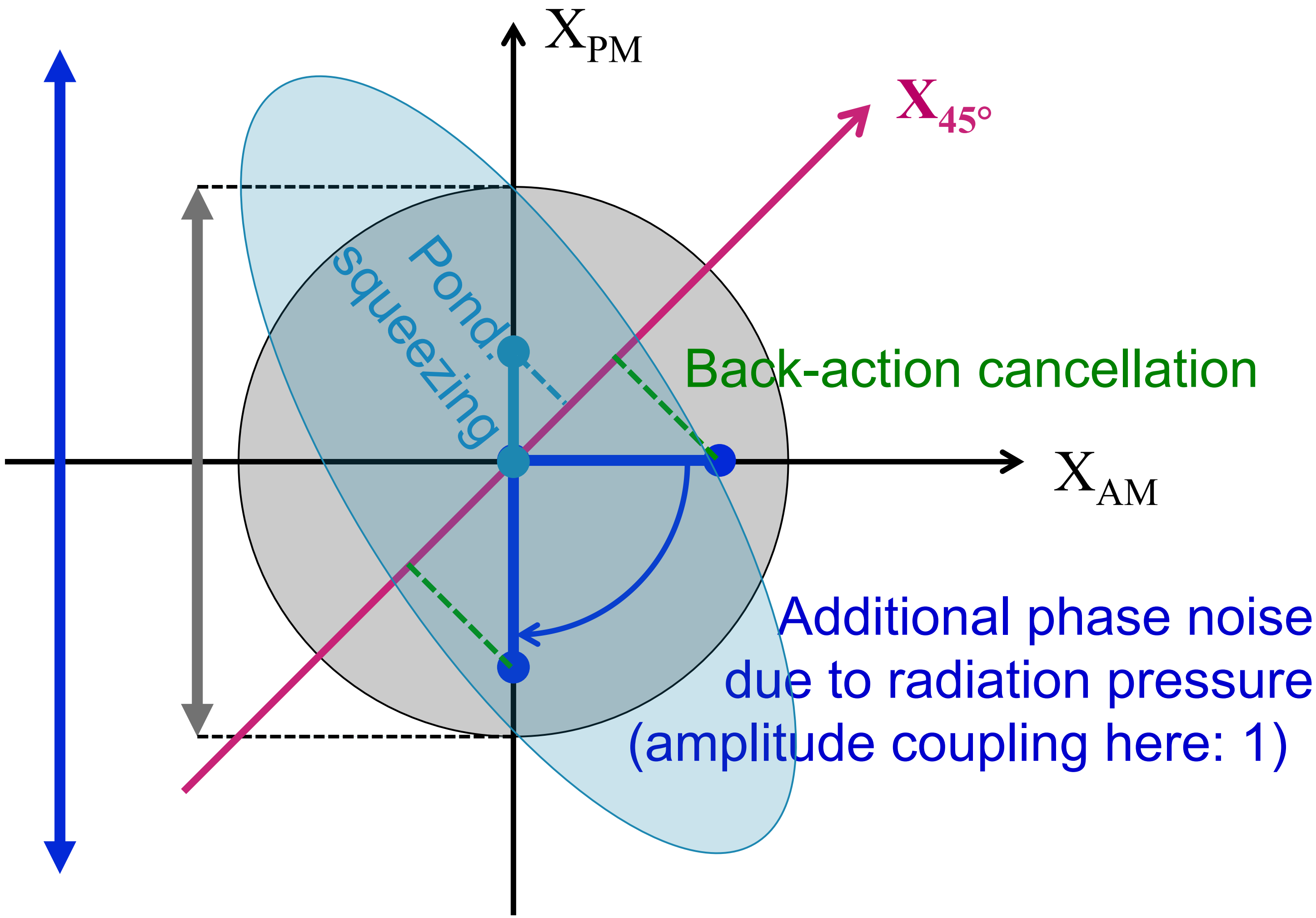
Squeezing and Full Evasion of RPN



Ponderomotive Squeezing



Ponderomotive Squeezing



Ponderomotive Squeezing

- Is of rather limited strength (requires RPN being dominant
- has a frequency dependent angle (and strength)
- good to cancel back-action, less good as a squeezing source

Recent experiments:

- Observation of radiation pressure noise on a membrane at NIST
  [T. P. Purdy, R. W. Peterson, C. A. Regal, Science **339**, 801 (2013)]

- Observation of ponderomotive squeezing at Caltech
  [A.H. Safavi-Naeini, S. Gröblacher, J.T. Hill, J. Chan, M. Aspelmeyer, O. Painter, arXiv:1302.6179]

Quantum noise in GW detectors

18/06/13

1) Photon counting noise (no real mirror displacement)

2) Radiation pressure noise
(momentum transfer from light to mirror)

(3) Zero-point fluctuation of the mirror
is <u>not</u> an issue!)

„Opto-mechanics does not need
an optical cavity"

Again: We are interested in Modulations of the light power!

1) Photon counting noise has a white spectrum!

„Shot noise (SN) per $\Delta\Omega = 1\,Hz$ equivalent to a variance of apparent mirror displacement $\Delta^2 x$ in m^2 for a simple Michelson interferometer"

$$\Delta^2 x_{SN} = \frac{\hbar c \cdot \lambda}{4\pi P_0} \equiv S_{x,SN} \quad \text{in} \left[\frac{m^2}{Hz}\right]$$

„Displacement normalized (single-sided) power spectral density"

2) Radiation pressure noise

Force : $|\vec{F}| = \frac{2 \cdot P_0}{c}$ ← retro-reflection; ← light power reflected

Quantum uncertainty

$$\Delta^2 P_0 = 2\hbar\omega P_0 \equiv S_{P_0,SN} \quad \text{in} \left[\frac{W^2}{Hz}\right]$$

$$\Rightarrow \quad S_{F,SN} = \left(\frac{2}{c}\right)^2 \cdot S_{P_0,SN} = \frac{8\hbar\omega P_0}{c^2} \quad \text{in} \left[\frac{N^2}{Hz}\right]$$

Independent of Fourier frequency Ω

$\hat{=}$ white spectrum.

The effect of the back-action force on mirror displacement (rpn)

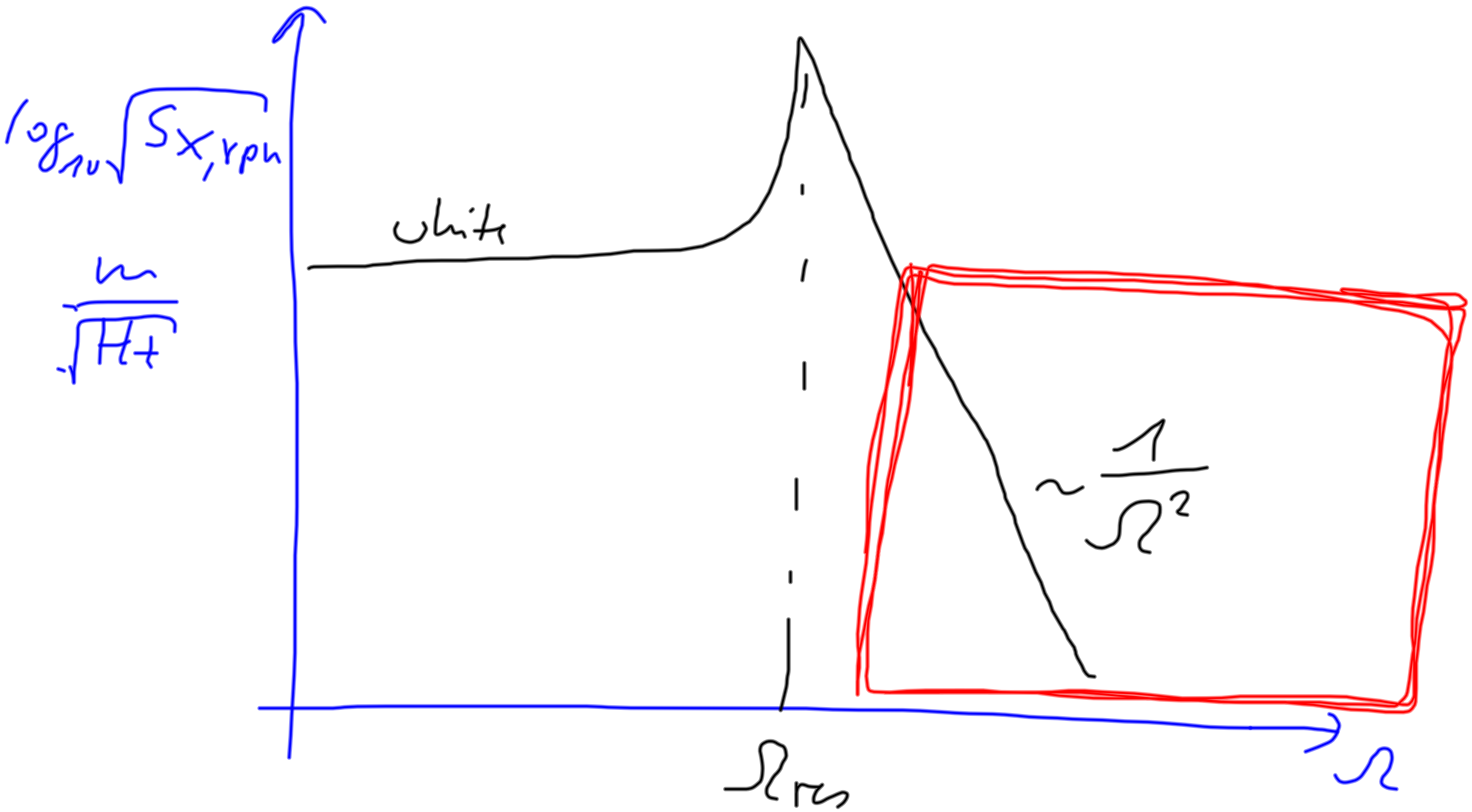
- effect is stronger the smaller the mass
- effect strong on mechanical resonance

$$S_{x,rpn} = |H(\Omega)|^2 \cdot S_{F,SN}$$

$$= \left| \frac{-1}{m(\Omega^2 + i\gamma\Omega - \Omega_{res}^2)} \right|^2 \cdot \frac{8\hbar\omega P_0}{c^2}$$

„Radiation pressure noise" in $\frac{m^2}{Hz}$

Back-action is not white



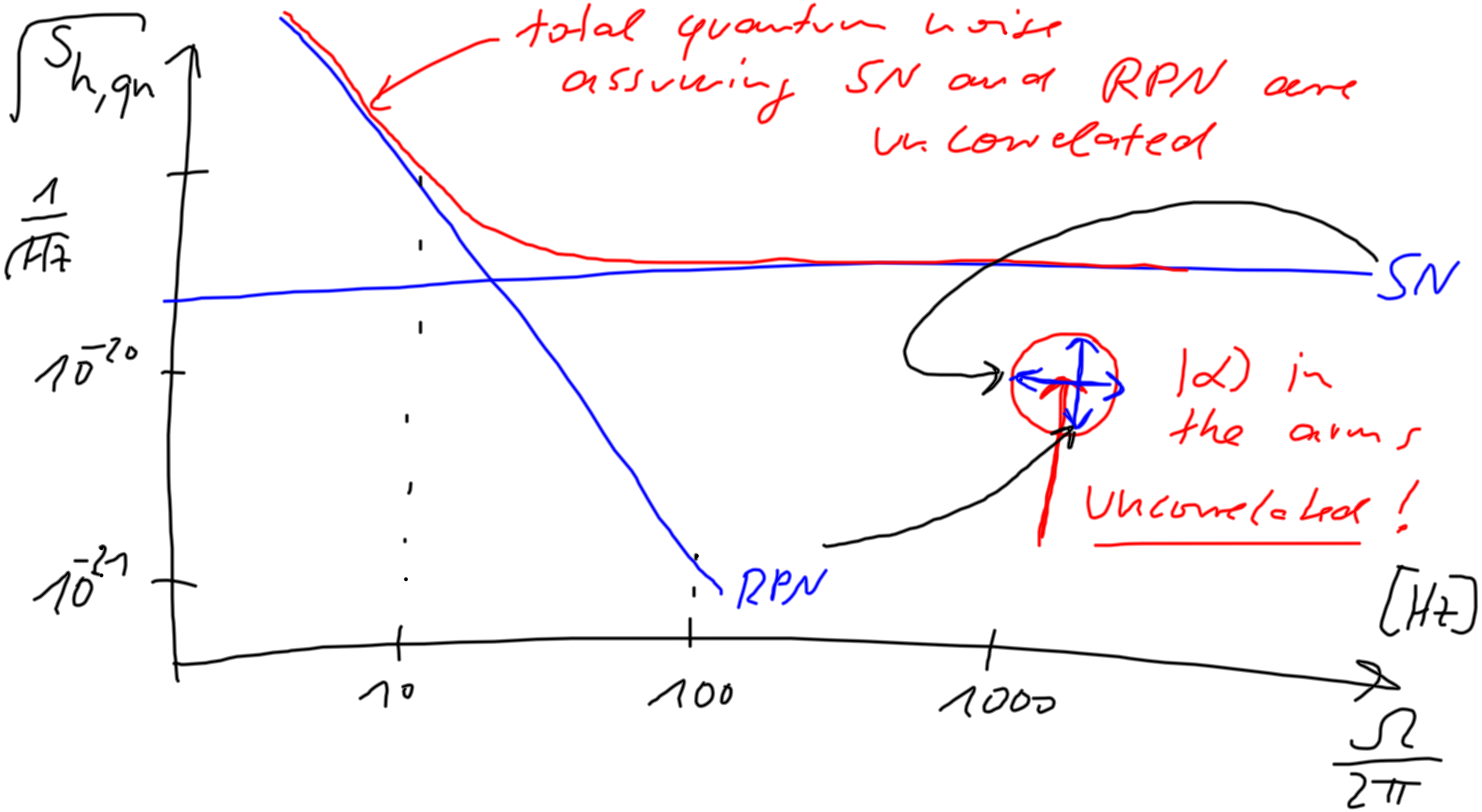
For $\Omega \gg \Omega_{res}$:

$$\sqrt{S_{x,rpn}} \simeq \frac{1}{m\Omega^2}\sqrt{\frac{8\hbar\omega P_0}{c^2}}$$

Recalibration: $\Delta h = \Delta x \frac{1}{L}$

$$\Rightarrow \sqrt{S_{h,rpn}} = \frac{1}{L \cdot m \cdot \Omega^2}\sqrt{\frac{8\hbar\omega P_0}{c^2}} \quad \text{in} \left[\frac{1}{\sqrt{Hz}}\right]$$

GWD Quantum Noise



The SQL for a force measurement

Assuming uncorrelated SN and RPN

→ add variances of the two and find the minimum when varying P_0

$$\left(\frac{\hbar c^2}{2\omega P_0} + \frac{8\hbar\omega P_0}{c^2 m^2 \Omega^2} \right) \Big| \min / P_0$$

} derivative

$$\rightarrow \quad P_{0,opt} = \frac{c^2 m \Omega^2}{4\omega}$$

depends on frequency

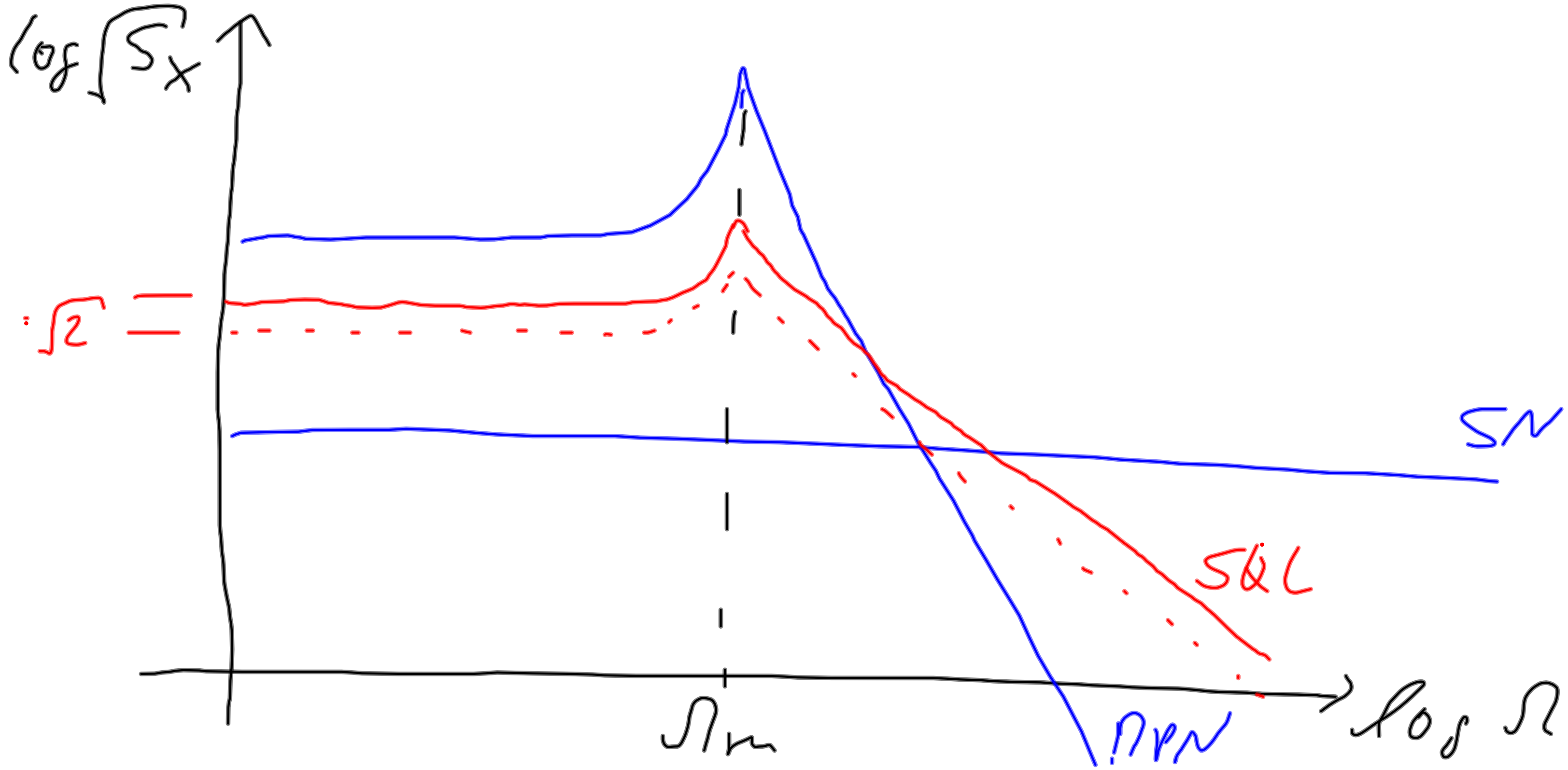
$c = \lambda \cdot \frac{\omega}{2\pi}$ laser wavelength

$\rightarrow$ $$S_{x,SQL} = \frac{4\hbar}{m\Omega^2}$$

Spectrum of SQL

$\hat{=}\; S_{x,SN} = S_{x,RPN}$

$= \frac{1}{2} S_{x,SQL}$



The SQL <u>can</u> be surpassed [Jaekel & R. 1990] using squeezed light having a frequency dependent squeezing angle.

Backaction can be evaded completely by using output filter cavities „variational output".

The latter is linked to *ponderomotive squeezing*.

See Powerpoint slides

— end —