Introduction to Optomechanics

Klemens Hammerer

Leibniz University Hannover
Institute for Theoretical Physics
Institute for Gravitational Physics (AEI)
Optomechanics

- **Cavity Optomechanics:** Mechanical degree of freedom coupled to a cavity mode

Mean energy in thermal equilibrium:
\[
\langle E \rangle = k_B T
\]

Can be used to **cool** the mirror:

\[
T_{\text{eff}} < T
\]

Can it be used to observe **quantum effects** with macroscopic systems?

“Quantum Optomechanics”
Optomechanical Systems

$\omega_m \approx \text{Hz} \ldots 10 \text{ GHz}$

$m_{eff} \approx 1 \text{ pg} \ldots 1 \text{ kg}$

D. Bowmeester, Santa Barbara/Leiden

LIGO – Laser Interferometer Gravitational Wave Observatory
Optomechanical Systems

Micromirrors
Aspelmeyer (Vienna)
Heidmann (Paris)
Bouwmeester (St Barbara, Leiden)

Micromembranes
Harris (Yale)
Kimble (Caltech)
Treutlein (Basel)

Microtoroids
Kippenberg (MPQ)
Weig (LMU)
Vahala (Caltech)
Bowen (UQ)
Optomechanical Systems

Optomechanical Crystals
Painter (Caltech)
Tang (Yale)

Optomechanics with BEC
Esslinger (Caltech)
Stamper-Kurn (Berkeley)

Levitated Nanoobjects
Theory:
Chang (Caltech)
Romero-Isart (MPQ, ICFO)
Experiment:
Raizen (Austin)
Mechanical Systems Coupled to Light

Quantum Optomechanics
M. Aspelmeyer, F. Marquardt, T. Kippenberg, RMP, arXiv:1303.0733

Macroscopic Quantum Mechanics: Theory and Experimental Concepts of Optomechanics
Y. Chen
arXiv:1302.1924
Quantum MECHANICAL Tests of the Superposition Principle

Marshall
Bouwmeester
Penrose
PRL 2003
Quantum Mechanical Tests of the Superposition Principle

• “Ramsey – Approach”

Create superposition → Evolution: Schrödinger Equ. & ?? → Measure

• in matter wave interferometry

K Hornberger, S Gerlich, P Haslinger, S Nimmrichter, M Arndt
RMP 84, 157 (2012)
Quantum Mechanical Tests of the Superposition Principle

• “Ramsey – Approach”

Create superposition → Evolution: Schrödinger Equ. & ?? → Measure

• in matter wave interferometry

H Müntinga
W Schleich
E Rasel et al.
PRL 110 093602 (2013)
Quantum Mechanical Tests of the Superposition Principle

- “Ramsey – Approach”

Create superposition  \[ \rightarrow \] Evolution: Schrödinger Equ. & ??

- in optomechanics with nanospheres

Romero-Isart et al.
PRL 107, 020405 (2011)
Quantum Mechanical Tests of the Superposition Principle

• Tests based on **Continuous Measurements**

  ![Diagram showing a continuous measurement process](image)

  • **idea:**
    - use continuous (precision) measurement to simultaneously prepare and measure superposition states
    - compare predictions according to (stochastic) Schrödinger equation with prediction from (Schrödinger equation & nonstandard model)

  *Macroscopic Quantum Mechanics: Theory and Experimental Concepts of Optomechanics*
  
  Y. Chen, arXiv:1302.1924

• **goal:** create and monitor pure quantum states (superposition states) of mechanical oscillators via continuous measurements
This talk

• Quantum Nondemolition Measurements of Mechanical Oscillators

• Continuous Measurement: measurement sensitivity & purity of conditional quantum state

• Quantum control via time continuous teleportation

tomorrow’s talk:

• Nonclassical superposition states via nonlinear dynamics

• Quantum control of optomechanical systems via coupling to atomic systems
Quantum Description

- **Mechanical oscillator**
  - creation/annihilation operator \([b, b^\dagger] = 1\)
  - Hamiltonian \(H = \omega_m b^\dagger b\)
  - quadrature operators 
    \[x_m = \frac{1}{\sqrt{2}} (b + b^\dagger)\]
    \[p_m = -\frac{i}{\sqrt{2}} (b - b^\dagger)\]
  - physical position and momentum 
    \(X_m = x_{ZPF} x_m\)
    \[x_{ZPF} = \sqrt{\frac{\hbar}{2m\omega_m}}\]

- **Cavity**
  - creation/annihilation operator \([a, a^\dagger] = 1\)
  - Hamiltonian \(H = \omega_c a^\dagger a\)
  - cavity frequency \(\omega_c = \frac{c\pi n}{L}\), \(n \in \mathbb{N}\)
  - Length depends on mirror position!
Quantum Description

- **cavity frequency for moving mirror**

\[ \omega_c(X_m) = \omega_c + \frac{\partial \omega_c}{\partial X_m} X_m = \omega_c - g_0 x_m \]

\[ g_0 = - \frac{\partial \omega_c}{\partial X_m} x_{ZPF} = \frac{\omega_c}{L} x_{ZPF} \]

- **Optomechanical Hamiltonian**

\[ H = \omega_m b^\dagger b + \omega_c(X_m) a^\dagger a \]
\[ = \omega_m b^\dagger b + \omega_c a^\dagger a - g_0 a^\dagger a x_m \]

\[ x_m = \frac{1}{\sqrt{2}} (b + b^\dagger) \]

- **Coupling rate** in a Fabry-Perot microcavity \( L \approx 10 \times \lambda \)

\[ g_0 \approx 10^{19} \text{ Hz/m} \times x_{ZPF} \]

\[ x_{ZPF} = \sqrt{\frac{\hbar}{2m\omega_m}} = 10^{-16} \text{ m} \]

for 1ng and 1MHz oscillator

\[ g_0 \approx \text{kHz} \]
Equations of motion

- **Hamiltonian**

\[ H = \omega_m b^\dagger b + \omega_c a^\dagger a - g_0 a^\dagger a x_m \]

- **Equations of motion**

**cavity**

\[ \dot{a} = -\left(\kappa + i\omega_c\right)a - ig_0 ax_m + E e^{-i\omega_L t} - \sqrt{2\kappa} a_{in} \]

**decay** **drive** **vacuum noise**

slowly varying variable \( \tilde{a}(t) = a(t) e^{i\omega_L t} \)

\[ \dot{\tilde{a}} = -\left(\kappa - i\Delta\right)\tilde{a} - ig_0 \tilde{a} x_m + E - \sqrt{2\kappa} \tilde{a}_{in} \]

\[ \Delta = \omega_L - \omega_c \]

laser detuning from cavity resonance
Equations of motion

- **Hamiltonian**

\[ H = \omega_m b^\dagger b + \omega_c a^\dagger a - g_0 a^\dagger ax_m \]

- **Equations of motion**

  **cavity**

\[ \dot{a} = -(\kappa - i\Delta)a - ig_0 ax_m + E - \sqrt{2\kappa} a_{in} \]

  decay  drive  vacuum noise

  **mechanical oscillator**

\[ \dot{x}_m = \omega_m p_m \]

\[ \dot{p}_m = -\omega_m x_m - \gamma p_m + g_0 a^\dagger a - \sqrt{2\gamma} f(t) \]

  decay  thermal force
Equations of motion

- **Quantum** equations of motion

\[
\begin{align*}
\dot{a} &= -(\kappa - i\Delta)a - ig_0ax_m + E - \sqrt{2}\kappa a_{in} \\
\dot{x}_m &= \omega_mp_m \\
\dot{p}_m &= -\omega_mx_m - \gamma p_m + g_0a^\dagger a - \sqrt{2}\gamma f_T(t)
\end{align*}
\]

- **Classical** equations of motion

\[
\begin{align*}
\dot{\alpha} &= -(\kappa - i\Delta)\alpha - ig_0\alpha x_m + E \\
\dot{x}_m &= \omega_mp_m \\
\dot{p}_m &= -\omega_mx_m - \gamma p_m + g_0|\alpha|^2 - \sqrt{2}\gamma f_T(t)
\end{align*}
\]

no vacuum fluctuations

classical random force
Equations of motion

- **Classical** equations of motion at zero temperature: (deterministic)

\[
\dot{\alpha} = -(\kappa - i\Delta)\alpha - ig_0\alpha x_m + E
\]

\[
\dot{x}_m = \omega_m p_m
\]

\[
\dot{p}_m = -\omega_m x_m - \gamma p_m + g_0|\alpha|^2
\]

Equations describe nonlinear, highly nontrivial physics: multistability, self induced oscillations, chaos…

F. Marquardt, M. Ludwig

Assume stable steady state solution exist with mean values \( \bar{\alpha}, \bar{x}_m, \bar{p}_m \)

- We want to analyze the **quantum** dynamics around these mean values due to classical random forces and quantum noise on mirror and cavity:

Introduce fluctuation operators:

\[
\delta a = a - \bar{\alpha}
\]

\[
[\delta a, \delta a^\dagger] = 1
\]

\[
\delta x_m = x_m - \bar{x}_m
\]

\[
[\delta x_m, \delta p_m] = i
\]

\[
\delta p_m = p_m - \bar{p}_m
\]
Equations of motion

• Fluctuations evolve according to

\[
\begin{align*}
\delta \dot{a} &= -(\kappa - i\Delta)\delta a - ig_0 \bar{\alpha} \delta x_m - ig_0 \delta a \delta x_m - \sqrt{2\kappa} a_{in} \\
\delta \dot{x}_m &= \omega_m \delta p_m \\
\delta \dot{p}_m &= -\omega_m \delta x_m - \gamma \delta p_m + g_0 \bar{\alpha} (\delta a + \delta a^\dagger) + g_0 \delta a^\dagger \delta a - \sqrt{2\gamma} f_T(t)
\end{align*}
\]

coupling strength of order: \( g_0 \bar{\alpha} = g \gg g_0 \)

for large intracavity amplitude \( \bar{\alpha} \gg 1 \)

drop small, nonlinear terms
Equations of motion

- **Quantum fluctuations** evolve according to **linearized** equations of motion

\[
\begin{align*}
\delta \dot{a} &= - (\kappa - i \Delta) \delta a - ig \delta x_m - \sqrt{2\kappa} \ a_{in} \\
\delta \dot{x}_m &= \omega_m \delta p_m \\
\delta \dot{p}_m &= - \omega_m \delta x_m - \gamma \delta p_m + g(\delta a + \delta a^\dagger) - \sqrt{2\gamma} f_T(t)
\end{align*}
\]

- **Classical description**: replace operators by C-numbers, drop vacuum noise

\[
\begin{align*}
\delta \dot{\alpha} &= - (\kappa - i \Delta) \delta \alpha - ig \delta x_m \\
\delta \dot{x}_m &= \omega_m \delta p_m \\
\delta \dot{p}_m &= - \omega_m \delta x_m - \gamma \delta p_m + g(\delta \alpha + \delta \alpha^*) - \sqrt{2\gamma} f_T(t)
\end{align*}
\]
Equations of motion

- Quantum fluctuations evolve according to linearized equations of motion

\[
\begin{align*}
\delta \dot{a} &= -(\kappa - i\Delta)\delta a - ig \delta x_m - \sqrt{2\kappa} a_{in} \\
\delta \dot{x}_m &= \omega_m \delta p_m \\
\delta \dot{p}_m &= -\omega_m \delta x_m - \gamma \delta p_m + g(\delta a + \delta a^\dagger) - \sqrt{2\gamma} f_T(t)
\end{align*}
\]

- Equations of motion in terms of quadratures:

\[
\begin{align*}
\delta \dot{x}_c &= -\Delta \delta p_c - \kappa \delta x_c - \sqrt{2\kappa} x_{c,in} \\
\delta \dot{p}_c &= \Delta \delta x_c - \kappa \delta p_c + \sqrt{2g} \delta x_m - \sqrt{2\kappa} p_{c,in} \\
\delta \dot{x}_m &= \omega_m \delta p_m \\
\delta \dot{p}_m &= -\omega_m \delta x_m - \gamma \delta p_m + \sqrt{2g} \delta x_c - \sqrt{2\gamma} f_T(t)
\end{align*}
\]

\[
\begin{align*}
\delta x_c &= \frac{1}{\sqrt{2}}(\delta a_c + \delta a_c^\dagger) \\
\delta p_c &= -\frac{i}{\sqrt{2}}(\delta a_c - \delta a_c^\dagger)
\end{align*}
\]
Equations of motion

• **Quantum fluctuations** evolve according to **linearized** equations of motion

\[
\begin{align*}
\delta \dot{a} &= -(\kappa - i\Delta)\delta a - ig \delta x_m - \sqrt{2\kappa} \ a_{in} \\
\delta \dot{x}_m &= \omega_m \delta p_m \\
\delta \dot{p}_m &= -\omega_m \delta x_m - \gamma \delta p_m + g(\delta a + \delta a^\dagger) - \sqrt{2\gamma} \ f_T(t)
\end{align*}
\]

• Equations of motion in terms of **amplitude (creation/annihilation) operators**:

\[
\begin{align*}
\delta \dot{a} &= -(\kappa - i\Delta)\delta a - \frac{ig}{\sqrt{2}} (\delta b + \delta b^\dagger) - \sqrt{2\kappa} \ a_{in} \\
\delta \dot{b} &= -(\gamma + i\omega_m)\delta b - \frac{ig}{\sqrt{2}} (\delta a + \delta a^\dagger) - \sqrt{2\gamma} \ f_T(t)
\end{align*}
\]

Corresponds to effective **optomechanical interaction Hamiltonian**

\[
H = -\Delta \delta a^\dagger \delta a + \omega_m \delta b^\dagger \delta b - \frac{g}{\sqrt{2}} (\delta a + \delta a^\dagger)(\delta b + \delta b^\dagger)
\]

RWA for noise: Valid for high Q Large T

compare to: \[H = g_0 a^\dagger ax_m\]
Radiation Pressure Interactions in Optomechanics

- (linearized) radiation pressure interaction in optomechanical systems

\[
H = g(\delta a + \delta a^{\dagger})(\delta b + \delta b^{\dagger}) = gX_L X_M
\]
Squeezing via Quantum Non Demolition Measurement

\[ X_L P_L \rightarrow X_M P_M \]
Squeezing via Quantum Non Demolition Measurement

\[ X_L P_L \]

\[ X_M P_M \]

\[ e^{-igtX_LX_M} \]

in

\[ P_L \]

\[ X_L \]

\[ X_M \]

out

\[ P_L \]

\[ \sim X_M \]

\[ \sim X_L \]
Squeezing via Quantum Non Demolition Measurement

\[ P_L \rightarrow p \]

\[ X_L P_L \]

\[ X_M P_M \]

in

\[ P_L \]

\[ X_L \]

\[ P_M \]

\[ X_M \]

out

\[ P_L \]

\[ X_L \]

\[ \sim X_M \]

\[ P_M \]

\[ X_M \]

\[ \sim X_L \]

conditioned on measurement

final

\[ P_L \]

\[ X_L \]

\[ p \]

\[ \sim p \]

squeezed state!
Squeezing via Quantum Non Demolition Measurement

- QND Hamiltonian
  \[ H = g X_L X_M \]

- Heisenberg picture
  \[ X_{L}^{out} = e^{iH\tau} X_L e^{-iH\tau} = X_L \]
  \[ P_{L}^{out} = P_L + \kappa X_M \]
  \[ X_{M}^{out} = X_M \]
  \[ P_{M}^{out} = P_M + \kappa X_L \]

- conditional atomic variance
  \[
  \Delta X_M^2 \bigg|_{P_L} = \Delta X_{M}^{out}^2 - \frac{\langle X_{M}^{out} P_{L}^{out} \rangle^2}{\Delta P_{L}^{out}^2}
  \]
  \[
  = \frac{1}{2} \left(1 - \frac{\kappa^2}{1 + \kappa^2}\right) = \frac{1}{2} \frac{1}{1 + \kappa^2} < \frac{1}{2}
  \]

KH, A.S. Sorensen, E.S. Polzik, RMP. 82, 1041 (2010);
QND Measurement with Short Pulses

- linearized radiation pressure interaction

\[ H = g X_L X_M \]

for short pulses \( \tau \ll \frac{1}{\omega_M} \)

- relevant decoherence rate of oscillator

\[ \tau_{coh}^{-1} = \gamma T = \gamma_M \bar{n} = \frac{k_B T}{\hbar Q_M} \]

requires
  - short pulses compatible with cavity
  - optimization of pulse profile
  - matching of LO profile to compensate for pulse distortion
  - good cavity design…

M. Vanner et al. PNAS 108, 16182 (2011)
Mechanical Squeezing via QND Measurement

• Application: Cooling via successive QND measurements


Experiment with musec pulses: M. Vanner et al. quant-ph/1211.7036

• Application: Improved position/force sensing in pulsed or stroboscopic measurements

Braginsky, Khalili...
Continuous Measurement and Back Action

- Full dynamics is not of QND type

\[ H = \frac{\omega_M}{2} (X_M^2 + P_M^2) + gX_LX_M = H_0 + H_1 \]

\[ [H_0, H_1] \neq 0 \]

- Cavity phase quadrature reads out position

\[ \dot{P}_L = gX_M \quad \text{+ cavity decay + vacuum noise} \]

Position evolves freely

\[ \dot{X}_M = \omega_M P_M \]

Momentum couples to amplitude quadrature (radiation pressure)

\[ \dot{P}_M = -\omega_M X_M + gX_L \quad \text{+ decay + thermal noise} \]

Amplitude noise couples via mechanics into phase quadrature: back action of light
Continuous Measurement Sensitivity

- phase quadrature in frequency domain

\[ P_{L}^{\text{out}}(\omega) = P_{L}^{\text{in}} + \frac{g}{\sqrt{\kappa}} X_{M} + \frac{g^{2}}{\kappa} \chi_{M} X_{L}^{\text{in}} \]

- noise spectral density referred to signal strength

\[ \frac{S_{P_{L}^{\text{out}}}(\omega)}{g^{2}/\kappa} = \frac{\kappa}{g^{2}} \left( S_{P_{L}^{\text{in}}} + \frac{g^{2}}{\kappa} S_{X_{M}} + \frac{g^{4}}{\kappa^{2}} |\chi_{M}|^{2} S_{X_{L}^{\text{in}}} \right) = \frac{\kappa}{g^{2}} + S_{X_{M}} + \frac{g^{2}}{\kappa} |\chi_{M}|^{2} \]

on resonance \( \omega = \omega_{M} \):

- sensitivity

\[ \text{sensitivity} = \frac{\kappa}{g^{2}} + \frac{2\bar{n} + 1}{\gamma_{M}} + \frac{g^{2}}{\kappa} \frac{1}{\gamma_{M}^{2}} \]

...mechanical susceptibility

\[ \chi_{M}(\omega) = \frac{1}{\omega - \omega_{M} - i\gamma_{M}} \]

\[ S_{X_{M}} = \chi_{M} \left( 2\bar{n} + 1 \right) \]

\[ \bar{n} \approx \frac{k_{B}T}{\hbar \omega_{M}} \]
Continuous Measurement Sensitivity

- sensitivity \[ \text{sensitivity} = \frac{\kappa}{g^2} + \frac{2\bar{n} + 1}{\gamma_M} + \frac{g^2}{\kappa} \frac{1}{\gamma_M^2} \]

- standard quantum limit:
  shot noise = back action noise
  achieved for power such that
  \[ \frac{g^2}{\kappa \gamma_M} = 1 \]

- back action larger than thermal noise
  achieved for power such that
  \[ C = \frac{g^2}{\kappa \gamma_M (2\bar{n} + 1)} = \frac{g^2}{\kappa \gamma_T} \geq 1 \]

  “strong optomechanical cooperativity”

\[ g = g_0 \alpha \quad g^2 \sim \text{power} \]

\[ S_{SSQL} \]

\[ P_{SSQL} \quad \frac{g^2}{\kappa} \sim \text{Power} \]

\[ \frac{g^2}{\kappa} \]

\[ C \geq 1 \]

\[ \gamma_T = \gamma_M \bar{n} = \frac{k_B T}{\hbar Q_M} \]

*Introduction to Quantum Noise, Measurement and Amplification*

Clerk, et al, RMP 82, 1155 (2010)
Strong Optomechanical Cooperativity

- Back action noise recently observed with micromechanical oscillators for the first time

\[
C = \frac{g^2}{\kappa \gamma T} \geq 1
\]

[Purdy, Peterson, Regal, Science 339, 801 (2013)]

[observed before with atomic oscillators
Murch, Stamper-Kurn, Nat. Phys. 4, 561 (2008)]

- optomechanical ground state cooling also requires \( C \geq 1 \)


Marquardt, Wilson-Rae
Strong Optomechanical Cooperativity

- Back action noise recently observed with micromechanical oscillators for the first time

\[ C = \frac{g^2}{\kappa \gamma T} \geq 1 \]


[observed before with atomic oscillators
Murch, Stamper-Kurn, Nat. Phys. 4, 561 (2008)]

- optomechanical cooperativity per single photon

\[ C_0 = \frac{C}{n_{\text{phot}}} = \frac{g^2}{\kappa \gamma T |\alpha|^2} = \frac{g_0^2}{\kappa \gamma T} \]

(linearized vs single photon coupling \( g = \alpha g_0 \))

M. Aspelmeyer, F. Marquardt, T. Kippenberg, RMP, arXiv:1303.0733
Conditional Quantum State

- Conditional quantum state according to stochastic master equation

\[
\frac{d\rho}{dt} = \{-i[H, \rho] + L_M \rho + L_c \rho\} dt + \mathcal{H}[ae^{i\phi}] \rho dW
\]

unconditional dynamics conditional dynamics

\[
\mathcal{H}[ae^{i\phi}] \rho dW = \sqrt{2\kappa} \left\{ e^{-i\phi} (a - \langle a \rangle) \rho + e^{i\phi} \rho (a^\dagger - \langle a^\dagger \rangle) \right\} dW
\]

solution: Monte Carlo

- For linearized interaction and homodyne detection of light: Gaussian states and dynamics

\[
H = \frac{\omega_M}{2} (X_M^2 + P_M^2) - \Delta a^\dagger a + g X_L X_M
\]

Stochastic Master Equation is equivalent to

- stochastic evolution of first moments \( \langle X_M(t) \rangle \) \( \langle P_M(t) \rangle \)

- deterministics evolution of second moments \( \Delta X_M^2 \) \( \Delta P_M^2 \) \( \langle X_M P_M + P_M X_M \rangle \)
Conditional Quantum State

- covariance matrix obeys Ricatti equation:
  \[ \gamma = \begin{pmatrix} \gamma_{xx} & \gamma_{xp} \\ \gamma_{xp} & \gamma_{pp} \end{pmatrix} \]
  \[ \dot{\gamma} = S\gamma + \gamma S^T + D + \gamma M \gamma \]
  \[ \gamma_{xx} = 2\Delta X_M^2 \]
  \[ \gamma_{pp} = 2\Delta P_M^2 \]
  \[ \gamma_{xp} = \langle X_M P_M + P_M X_M \rangle \]

  drift matrix      diffusion matrix
  unconditional dynamics  conditional dynamics

- stochastic (but known) mean values and deterministic covariance matrix fully determine quantum state, e.g. Wigner function etc

  purity of state  =  \text{tr}[\rho^2] = \frac{1}{\sqrt{\text{Det}\gamma}}

  uncertainty product  \[ \Delta X_M \Delta P_M = \frac{1}{2} \sqrt{\frac{2 + C}{C}} \to \frac{1}{2} \]

  pure state!
  even for  \[ \bar{n} \to \infty \]

Belavkin et al., arXiv:0506018
Genoni et al., PRA 87 042333 (2013)
Vasilyev, Muschik, KH, PRA, arXiv:1303.5888

Quantum Optics in Phase Space
W. Schleich
Continuous Preparation and Monitoring of Pure Quantum States

- the conditional state is pure, Gaussian

- position and momentum are effectively probed with equal sensitivity if in this case the conditional quantum state will be a **coherent** state

- a **squeezed** state is generated if

\[
P_{\text{out}}^L \quad |\Psi(t)\rangle
\]

\[
C \geq 1
\]

\[
\frac{g^2}{\kappa \omega_M} \leq \omega_M
\]

\[
\mathcal{D} = \frac{g^2}{\kappa \omega_M} = C \frac{\bar{n}}{Q} \geq 1
\]

per single photon

\[
\mathcal{D}_0 = \frac{\mathcal{D}}{|\alpha|^2} = \frac{g_0^2}{\kappa \omega_M}
\]

\[
D_0
\]

arXiv:1303.0733
Universal Quantum Control via Measurement and Feedback

- **idea:** (superposition) state engineering via continuous quantum teleportation

- drive optomechanical system on upper sideband

  \[
  H = \omega_M b_M^\dagger b_M - \Delta a_L^\dagger a_L + g(a_L + a_L^\dagger)(b_M + b_M^\dagger)
  \]

  \[\simeq g(a_L b_M + a_L^\dagger b_M^\dagger)\]

  entangling interaction between light & oscillator

- perform Bell measurement on light

- linear feedback transfers quantum statistics of light on oscillator:

  non-Gaussian statistics of light generates non-Gaussian state of oscillator

*pulsed* teleportation protocols:  
Romero-Isart, Pflanzer, Cirac, PRA 83, 013803 (2011)  
Hofer et al PRA 84, 052327 (2011)
Continuous Bell measurement with Gaussian Input Field

• linear interaction of arbitrary system with light field

\[ H_{\text{int}} = s a^\dagger(t) + s^\dagger a(t) \]

• conditional master equation for system under continuous Bell measurement

\[ \frac{d\rho}{dt} = i [H_{\text{sys}} + (1/4) \{(F_+ + iF_-)s + s^\dagger(F_+ - iF_-)\}, \rho] \\
+ (1/2) \{D[s - iF_+]\rho + D[s - F_-]\rho + w_3D[F_+ + F_-]\rho \\
+ (w_1 - w_3 - 1)D[F_+]\rho + (w_2 - w_3 - 1)D[F_-]\rho \} \]

measurement terms depend on |Ψ⟩

• linear feedback with “forces”

\[ [\dot{\rho}]_{\text{fb}} = -iI_+[F_+ , \rho] - iI_-[F_- , \rho] \]

• unconditional feedback master equation

\[ \dot{\rho} = -i \left[ H_{\text{sys}} + (1/4) \{(F_+ + iF_-)s + s^\dagger(F_+ - iF_-)\}, \rho \right] \\
+ (1/2) \{D[s - iF_+]\rho + D[s - F_-]\rho + w_3D[F_+ + F_-]\rho \\
+ (w_1 - w_3 - 1)D[F_+]\rho + (w_2 - w_3 - 1)D[F_-]\rho \} \]

Hofer, Vasilyev, Aspelmeyer, KH, arXiv:1303.4976
Continuous Quantum Teleportation

• select an entangling interaction by tuning to the upper sideband
  \[ H = g(a_L b_M + a_L^{\dagger} b_M^{\dagger}) \]
  i.e. \( s = b_M^{\dagger} \)

• feedback master equation (ideally)
  \[ \dot{\rho} = 2J\rho J^{\dagger} - J^{\dagger}J\rho - \rho J^{\dagger}J \]

with jump operator \( J|\Psi\rangle = 0 \) such that \( \rho(t) \rightarrow |\Psi\rangle\langle\Psi| \)

• including thermal noise & counterrotating terms:

Hofer, Vasilyev, Aspelmeyer, KH, arXiv:1303.4976

input squeezing - 6dB
Continuous Quantum Teleportation & Entanglement Swapping

- more general (non Gaussian) states: use field emitted from a second, source system and transfer to target system

field emitted by an cavity QED is a (continuous) Matrix Product State $|\Psi\rangle$ of the continuous 1D EM field: Barrett, KH et al. PRL 110, 090501 (2013)

- if both systems have an entangling interaction with light:
  time continuous entanglement swapping generates EPR state of oscillators

$$\dot{\rho}_{12} = 2J\rho_{12}J^\dagger - J^\dagger J\rho_{12} - \rho_{12}J^\dagger J$$

$$J_{12}|EPR\rangle = 0$$

generates entangled state of micromechanical oscillators in steady state

- entangled states with EPR squeezing $\sim C^{-1}$

superposition size scaling as $\sim C$

Hofer, Vasilyev, Aspelmeyer, KH, arXiv:1303.4976
Continuous Quantum Teleportation & Entanglement Swapping

- can also be applied to spins

\[ H_{\text{int}} = s \ a^\dagger(t) + s^\dagger a(t) \]

\[ s_i = \sqrt{z(1+z)}\sigma_i^+ - \sqrt{1-z}\sigma_i^- \quad (i = 1, 2; \ z \in [0, 1]) \]

for correct feedback steady state is \( |\Psi\rangle = |00\rangle - z|11\rangle \)

- including transmission losses
Continuous Quantum Teleportation & Entanglement Swapping

• setup is similar to a GWD Michelson interferometer: entangled state of mirror test masses predicted in

Miao, PRA 81 012114 (2010)

• continuous test of quantum mechanics with macroscopic objects:
  – parametrize how the dynamics of the object would deviate from standard quantum mechanics (for a given collapse model)
  – monitor true dynamics and exclude parameters

Macroscopic Quantum Mechanics: Theory and Experimental Concepts of Optomechanics
Y. Chen, arXiv:1302.1924
Requirements & Summary

• Experiments need to have a strong optomechanical cooperativity

\[ C = \frac{g^2}{\kappa \gamma T} \geq 1 \]

• the conditional state predicted by the stochastic master equation has to be reconstructed from measured photocurrent via Kalman/Wiener–filtering

![Diagram of optomechanical system](image)

applied to optomechanical system in free space \((C \ll 1)\)  

Furusawa et al 1305.0066

• continuous measurement at the quantum limit is
  – getting feasible in state of the art optomechanical systems
  – toolbox for universal quantum state engineering
  – detector of non-standard physics based on macroscopic superposition states (alternative to Ramsey-like approaches)
Gaussian States and Interactions

• The **optomechanical toolbox**
  
  – linearized interaction: \( H = -\Delta \delta a^\dagger \delta a + \omega_m \delta b^\dagger \delta b - \frac{g}{\sqrt{2}} (\delta a + \delta a^\dagger)(\delta b + \delta b^\dagger) \)
  
  – homodyne detection of light: measurement of \( \delta x_c, \delta p_c \)
  
  – injection of squeezed, coherent (Gaussian) light preserves Gaussian character of states.

• We can select the **dominant interaction** by choosing the detuning

\[
\begin{align*}
\text{red} & \quad \omega_L \\
\delta a^\dagger \delta b + \delta a \delta b^\dagger & \quad \text{cooling, state exchange} \\
\text{blue} & \quad \omega_L \\
\delta a \delta b + \delta a^\dagger \delta b^\dagger & \quad \text{entanglement} \\
\text{resonant} & \quad \omega_L \\
(\delta a + \delta a^\dagger)(\delta b + \delta b^\dagger) & \quad \text{displacement detection, QND measurement}
\end{align*}
\]
Non Gaussian States and Interactions

• How can we leave the Gaussian world?

• The Non Gaussian toolbox
  – inject Non-Gaussian light
    U. Akram, N. Kiesel, M. Aspelmeyer, G. J. Milburn, arxiv:1002.1517
  – photon counting (instead of homodyne detection)
  – non-bilinear Hamiltonians
    \[ H = g_1 a^\dagger a b^\dagger b \]
    can be engineered with micromembranes
    \[ H = g_0 a^\dagger a x_m \]
    radiation pressure interaction is fundamentally nonlinear, but notoriously weak: \( g = \bar{\alpha} g_0 \gg g_0 \)
Quantum Optomechanics

- Radiation pressure Hamiltonian is nonlinear

\[ H_{\text{int}} = \omega_c(x_m)a^\dagger a \simeq \omega_c a^\dagger a - g_0 a^\dagger a(b + b^\dagger) \]

For sufficiently strong coupling per single photon \( g_0 \) the dynamics will be Non-Gaussian.

- Do Non-Gaussian features persist in steady state? Negative Wigner Functions?

Master Equation

\[ \dot{\rho} = (L_m + L_c + L_{\text{int}})\rho \]

\[ L_m\rho = -i \left[ \omega_m b^\dagger b, \rho \right] + \gamma_m (\bar{n} + 1) D[b]\rho + \gamma_m \bar{n} D[b^\dagger]\rho, \]

thermal contact

\[ L_c\rho = -i \left[ -\Delta a^\dagger a - iE (a - a^\dagger), \rho \right] + \kappa D[a]\rho \]

driving field
cavity decay

\[ L_{\text{int}}\rho = -i \left[ -g_0 a^\dagger a (b + b^\dagger), \rho \right]. \]

\[ D[A]\rho = 2A\rho A^\dagger - A^\dagger A\rho - \rho A^\dagger A \]
Steady State

- Solve master equation for steady state \((L_m + L_c + L_{\text{int}}) \rho_{ss} = 0\) for (rather extreme) parameters

\[g_0 = 0.36 \omega_m \quad \kappa = 0.3 \omega_m \quad \gamma_m \simeq 10^{-3} \omega_m \quad E \simeq 0.3 \omega_m \quad T = 0\]

- Wigner function
Steady State

• Solve master equation for steady state \((L_m + L_c + L_{int}) \rho_{ss} = 0\) for (rather extreme) parameters

\[
\begin{align*}
g_0 &= 0.36 \omega_m \quad \kappa = 0.3 \omega_m \\
\gamma_m &\approx 10^{-3} \omega_m \quad E \approx 0.3 \omega_m \quad T = 0
\end{align*}
\]

• Wigner function

Qian, Clerk, KH, Marquardt
PRL 109, 253601, 2012
Steady State

- Solve master equation for steady state \((L_m + L_c + L_{int}) \rho_{ss} = 0\)
  for (rather extreme) parameters
  \[ g_0 = 0.36\omega_m \quad \kappa = 0.3\omega_m \quad \gamma_m \simeq 10^{-3}\omega_m \quad E \simeq 0.3\omega_m \quad T = 0 \]

- Wigner function

\[ \text{Wigner function} \]

\[ \text{Phonon Number } n_b \]

\[ \text{detuning } \Delta \]

\[ \text{oscillator } \omega_L, \omega_c \]

Qian, Clerk, KH, Marquardt
PRL 109, 253601, 2012
Steady State

- Solve master equation for steady state \( (L_m + L_c + L_{\text{int}}) \rho_{ss} = 0 \) for (rather extreme) parameters

\[
g_0 = 0.36 \omega_m \quad \kappa = 0.3 \omega_m \quad \gamma_m \simeq 10^{-3} \omega_m \quad E \simeq 0.3 \omega_m \quad T = 0
\]

- Wigner function
Steady State

- Wigner functions
- Phonon number and Fano factor

Qian, Clerk, KH, Marquardt
PRL 109, 253601, 2012
Limit Cycles

• Quantum equations of motion

\[ \dot{a} = \left[ i(\Delta + g_0(b + b^\dagger)) - \kappa \right] a + E + \text{noise} \]

\[ \dot{b} = -(i\omega_m + \gamma_m)b - ig_0a^\dagger a + \text{noise} \]

• limit cycles are classical nonlinear dynamics

\[ a \rightarrow \alpha \quad b \rightarrow \beta \]
Limit Cycles

• Classical equations of motion

\[ \dot{\alpha} = [i(\Delta + g_0(\beta + \beta^*)) - \kappa] \alpha + E \]

\[ \dot{\beta} = -(i\omega_m + \gamma_m)\beta - ig_0|\alpha|^2 \]

• slowly varying amplitude \( \tilde{\beta} = \beta e^{i\omega_m t} \)

\[ \dot{\alpha} = \left[ i(\Delta + g_0(\tilde{\beta} e^{i\omega_m t} + \text{c.c.}) - \kappa) \right] \alpha + E \]

solution \( \alpha(\tilde{\beta}, t) \approx \sum_{n=-\infty}^{\infty} \alpha_n e^{-in\omega_m t} \)

intensity

\[ |\alpha|^2 = \sum_n \alpha_{n-1}^* \alpha_n e^{-i\omega_m t} + \text{off resonant terms} \]

Marquardt, Ludwig…
RMP, arXiv:1303.0733
Limit Cycles

- mechanical amplitude

\[ \dot{\beta} = -(i\omega_m + \gamma_m)\beta - ig_0|\alpha|^2 \]

\[ \simeq - [i(\omega_m + \omega_{opt}(\beta)) + \gamma_m + \gamma_{opt}(\beta)] \beta \]

- nonlinear optical damping

\[ \gamma_{opt}(|\beta|) = \text{Re} \sum_n \frac{\alpha^*_n(|\beta|) \alpha_n(|\beta|)}{|\beta|} \]

\[ \Delta = 0.8\omega_m \]

\[ \frac{\gamma_{opt}}{\gamma_m} \]

no limit cycles for \( \Delta > 0 \)!

- negative Wigner functions?
Conditional state

- Conditional state as calculated from one trajectory of Monte-Carlo simulation

Loerch, KH, in prep
Collaborators:
M. Aspelmeyer & company
M.S. Kim, G.J. Milburn, C. Brukner, I. Pikovski
T. Osborne, S. Harrison, S. Barrett, T. Northup
C. Muschik

Group:
Sebastian Hofer  arXiv:1303.4976
Denis Vasilyev
Niels Loerch
Sergey Tarabrin

Thank you!

Support through:
DFG (QUEST), EC (MALICIA, iQUOEMS)

Winter School on Rydberg Physics and Quantum Information
Obergurgl Feb 10-14 1013  www.rqi.uni-hannover.de