Cavity optomechanics in new regimes and with new devices

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2. Single-photon regime
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Introduction
Cavity optomechanics

辐射压力力

腔模

机械振荡器
Cavity optomechanics: the mechanical oscillator

\[ \hat{H} = \frac{\hat{p}^2}{2m} + \frac{m\omega_M^2 \hat{x}^2}{2} \]

\[ \hat{x} = x_{ZPF}(\hat{b} + \hat{b}^\dagger) \]

\[ x_{ZPF} = \sqrt{\frac{\hbar}{2m\omega_M}} \]

\[ \hat{H} = \hbar\omega_M \left( \hat{b}^\dagger \hat{b} + \frac{1}{2} \right) \]

phonons

thermal state

\[ P_n \approx e^{-n/n_{th}} \]

with \[ n_{th} = \frac{k_B T}{\hbar\omega_M} \]
Cavity optomechanics: the cavity mode

\[ \hat{H} = \hbar \omega_C \left( \hat{a}^{\dagger} \hat{a} + \frac{1}{2} \right) \]

photons

\[ L = n \frac{\lambda}{2} \]

\[ f = \frac{c}{\lambda} = n \frac{c}{2L} \quad \omega_C = 2\pi f = \frac{n \pi c}{L} \]
Cavity optomechanics: the optomechanical coupling

\[ \hat{\alpha} \]

\[ L \]

\[ \omega_C(0) = \frac{n\pi c}{L} \equiv \omega_R \]

\[ L + x \]

\[ \omega_C(x) = \frac{n\pi c}{L + x} \]

\[ \frac{\omega_C(x)}{\omega_C(0)} = \frac{L}{L + x} \approx \left(1 - \frac{x}{L}\right) \]
\[ \hat{H} = \hbar \omega_C (\hat{x}) \hat{a}^{\dagger} \hat{a} + \hbar \omega_M \hat{b}^{\dagger} \hat{b} \]

with \[ \omega_C(x) = \left(1 - \frac{x}{L}\right) \omega_R \]

\[ \hat{H} = \hbar \left(1 - \frac{\hat{x}}{L}\right) \omega_R \hat{a}^{\dagger} \hat{a} + \hbar \omega_M \hat{b}^{\dagger} \hat{b} \]

\[ \hat{H} = \hbar \omega_R \hat{a}^{\dagger} \hat{a} + \hbar \omega_M \hat{b}^{\dagger} \hat{b} + \hbar g (\hat{b} + \hat{b}^{\dagger}) \hat{a}^{\dagger} \hat{a} \]

\[ \hat{x} = x_{\text{ZPF}} (\hat{b} + \hat{b}^{\dagger}) \text{ with } g = -x_{\text{ZPF}} \omega_R / L \]

N.B. We neglect the dynamical Casimir effect.
The field of optomechanics deals with systems in which optical and mechanical degrees of freedom are coupled to each other. Interaction between light and mechanical elements was already considered centuries ago by Johannes Kepler, who studied the tails of comets. Nowadays it is well known that light exerts a radiation pressure force on objects. The momentum of the impinging light is transferred to the mechanical object and thus influences its motion. Although the action of sunlight on comets looks spectacular and impressive, the force due to a single photon is small compared to the inertia of macroscopic objects. Nevertheless, radiation pressure has to be taken into account for precision measurements and is a major issue e.g. for the detection of gravitational waves.

To make use of the radiation pressure force, it is convenient to use cavities to enhance the light intensity and thereby the coupling strength. A typical optomechanical setup consists of a cavity with one fixed and one moveable end-mirror, see Fig. 0.1 (a).

Figure 0.1.: (a) Typical optomechanical system implementing dispersive coupling between optics and mechanics. (b) Schematic picture illustrating the general setup envisioned for dispersively coupled systems, also including coupling to the environment.

with the dimensionless parameters

- $\omega_m / \kappa$ good/bad-cavity limit
- $g / \kappa$ cavity shift per phonon in units of line width
- $\Omega / \kappa$ drive strength & detuning
- $\Delta / \omega_m$ detuning
- $\omega_m / \gamma$ mechanical quality factor
- $g / \omega_m$ oscillator displacement per photon in units of its ZPF

$n_{th}$ mechanical bath

optical bath

optomechanical interaction
Single-photon regime
The single-photon strong-coupling regime

\[ \hat{H} = \omega_R \hat{a}^\dagger \hat{a} + \omega_M \hat{b}^\dagger \hat{b} + g(\hat{b} + \hat{b}^\dagger) \hat{a}^\dagger \hat{a} \]

with \( g = -x_{\text{ZPF}} \omega_R / L \)

\[ \frac{g}{\omega_M} \sim 2, \quad \frac{\omega_M}{\kappa} \sim 0.1 \]


\[ \frac{g}{\kappa} \sim 0.007 - 0.1, \quad \frac{\omega_M}{\kappa} \sim 30 \]

Chan et al., Nature 478, 89 (2011)

\[ \frac{g}{\omega_M} \sim 5 \times 10^{-6}, \quad \frac{\omega_M}{\kappa} \sim 50 \]

Teufel et al., Nature 475, 359 (2011)

What happens when \( \frac{g}{\omega_M} \sim 1 \) and \( \frac{\omega_M}{\kappa} \sim 1 \)?
Single-photon regime

\[ \hat{H} = \omega_R \hat{a}^\dagger \hat{a} + \omega_M \hat{b}^\dagger \hat{b} + g(\hat{b} + \hat{b}^\dagger) \hat{a}^\dagger \hat{a} \]

Completing the square

\[ \hat{H} = \omega_R \hat{a}^\dagger \hat{a} + \omega_M \left( \hat{b} + \frac{g}{\omega_M} \hat{a}^\dagger \hat{a} \right)^\dagger \left( \hat{b} + \frac{g}{\omega_M} \hat{a}^\dagger \hat{a} \right) - \frac{g^2}{\omega_M} (\hat{a}^\dagger \hat{a})^2 \]

new equilibrium position of mechanical oscillator

\[ \hat{U} = \exp\left[ (\hat{b}^\dagger - \hat{b})(g/\omega_M) \hat{a}^\dagger \hat{a} \right] \]

polaron transformation

\[ \hat{U}^\dagger \hat{b} \hat{U} = \hat{b} + \frac{g}{\omega_M} \hat{a}^\dagger \hat{a} \]

\[ \hat{U}^\dagger \hat{a} \hat{U} = \hat{a} \exp\left[ (\hat{b}^\dagger - \hat{b})(g/\omega_M) \right] \]

\[ \hat{U}^\dagger \hat{H} \hat{U} = \left( \omega_R - \frac{g^2}{\omega_M} \hat{a}^\dagger \hat{a} \right) \hat{a}^\dagger \hat{a} + \omega_M \hat{b}^\dagger \hat{b} \]

Interaction is diagonal, but drive and damping are not!
Single-photon regime

\[ \hat{H} = \omega_R \hat{a}^{\dagger} \hat{a} + \omega_M \hat{b}^{\dagger} \hat{b} + g(\hat{b} + \hat{b}^{\dagger})\hat{a}^{\dagger} \hat{a} \]

Photon number conserved \[ [\hat{a}^{\dagger} \hat{a}, \hat{H}] = 0 \]

At fixed photon number \( n \), the mechanical oscillator is displaced by \( n x_0 \)

\[ -n x_0 / x_{ZPF} = -2ng / \omega_M \]

oscillator displacement per photon in units of its ZPF

The energy spectrum is

\[ E_{nm} = \omega_R n - g^2 n^2 / \omega_M + \omega_M m \]
Single-photon regime: the cavity response

In the single-photon regime multiple resonances appear.

\[ \omega_M / \kappa = 2, \omega_M / \gamma = 20, n_0 = 4 \Omega^2 / \kappa^2 \]

→ solution of quantum Langevin equations

Nunnenkamp et al., PRL 107, 063602 (2011)
also: Rabl, PRL 107, 063601 (2011)
Single-photon regime: the cavity output spectrum

\[ S(\omega) = \int_{-\infty}^{\infty} dt \, e^{i\omega t} \langle \hat{a}^\dagger(t) \hat{a}(0) \rangle \]

\[ S(\omega) / n_0 \]

\[ \omega / \omega_M \]

\[ n_{th} = 0 \]

\[ g = \omega_M \]

\[ \Delta = 0 \]

\[ \omega_R \]

\[ g^2 / \omega_M \]

\[ \omega_M \]

\[ x / x_{ZPF} \]

\[ 2g / \omega_M \]

\[ 2g / \omega_M \]

\[ \rightarrow \text{nonlinear photon-phonon coupling} \]

In the single-photon regime multiple sidebands appear.

\[ \omega_M / \kappa = 2, \omega_M / \gamma = 20, n_0 = 4\Omega^2 / \kappa^2 \]

\[ \rightarrow \text{solution of quantum Langevin equations} \]

Nunnenkamp et al., PRL 107, 063602 (2011)

also: Liao et al., arXiv:1201.1696
Dissipative coupling
Dispersive vs. dissipative optomechanics

Dispersive coupling

\[ \omega_C(\hat{x}) = \omega_R + \frac{\partial \omega_C}{\partial x} \hat{x} \]

Dissipative coupling

\[ \kappa(\hat{x}) = \kappa + \frac{\partial \kappa}{\partial x} \hat{x} \]

Elste et al., PRL 102, 207209 (2009)
Xuereb et al., PRL 107, 213604 (2011)
Sideband cooling: the quantum noise approach

At weak coupling all you need to know is the force spectrum

\[ \hat{H}_{\text{int}} = -\hat{F} \hat{x} \quad S_{FF}(\omega) = \int dt \, e^{i\omega t} \left\langle \hat{F}(t)\hat{F}(0) \right\rangle \]

Obtain the rates with Fermi’s Golden Rule

\[ \Gamma_{\text{opt}}^{\downarrow} = \frac{x_{ZPF}^2}{\hbar^2} S_{FF}(\omega_M) \quad \Gamma_{\text{opt}}^{\uparrow} = \frac{x_{ZPF}^2}{\hbar^2} S_{FF}(\omega_M) \]

And calculate the steady-state phonon number

\[ \bar{n}_M = \frac{\Gamma_M \bar{n}_{\text{th}} + \Gamma_{\text{opt}} \bar{n}_M^O}{\Gamma_M + \Gamma_{\text{opt}}} \]

\[ \bar{n}_M^O = \frac{\Gamma_{\text{opt}}^{\uparrow}}{\Gamma_{\text{opt}}} \]

\[ \Gamma_{\text{opt}} = \Gamma_{\text{opt}}^{\downarrow} - \Gamma_{\text{opt}}^{\uparrow} \]

optical damping

minimal phonon number

Clerk et al., RMP 82, 1155 (2010)

Marquardt et al., PRL 99, 093902 (2007)

Wilson-Rae et al., PRL 99, 093901 (2007)
Sideband cooling: the quantum noise approach

Calculate the force (i.e. shot-noise) spectrum

\[
S_{FF}(\omega) = \int dt \, e^{i\omega t} \left\langle \hat{F}(t)\hat{F}(0) \right\rangle \quad \text{with} \quad \hat{F} = \frac{\omega R}{L} \hat{a}^\dagger \hat{a}
\]

\[
S_{FF}(\omega) = \left( \frac{\hbar \omega_R}{L} \right)^2 \bar{n}_p \frac{\kappa}{(\omega + \Delta)^2 + (\kappa/2)^2}
\]

\[
\Delta = -\omega_M
\]

Ground-state cooling is possible in the sideband-resolved regime \( \omega_M \gg \kappa \)

Clerk et al., RMP 82, 1155 (2010)

Marquardt et al., PRL 99, 093902 (2007)
Wilson-Rae et al., PRL 99, 093901 (2007)
Fano resonance in the force spectrum

\[ \hat{H}_{\text{int}} = -\hat{F} \hat{x} \]

with \( \hat{F} \propto \sqrt{\kappa} \sum_q (\hat{a}_q \hat{a}_q^\dagger + \hat{a}_q^\dagger \hat{a}_q) \)

Linear theory with two noise sources!

\[ \hat{F} = \hat{F}_1 + \hat{F}_2 \]

\[ \hat{F}_1 \propto \hat{d}_{\text{in}} + \hat{d}_{\text{in}}^\dagger \]

\[ \hat{F}_2 \propto \hat{d} + \hat{d}^\dagger \]

They are random, but not independent!

\[ \hat{d}(\omega) \propto \chi_R(\omega) \hat{d}_{\text{in}} \]

\[ S_{FF}(\omega) \propto \left| 1 - \left( \frac{\kappa}{2} + i\Delta \right) \chi_R(\omega) \right|^2 \]

with \( \chi_R(\omega) = \left[ \frac{\kappa}{2} - i(\omega + \Delta) \right]^{-1} \) → Fano line shape

This enables ground state cooling outside the resolved-sideband limit!

Elste et al., PRL 102, 207209 (2009)
For dissipative coupling there are two cooling regions cooling and two instability regions.
Normal-mode splitting in strong-coupling regime

\[ S_{cc} (\omega) = \int dt \, e^{i \omega_t} \langle \hat{c}^\dagger (t) \hat{c} (0) \rangle \]

\[ \Delta = -\omega_M \]

Fano interference

Regions of instability

Normal-mode splitting

Weiss, Bruder, and Nunnenkamp, arXiv:1211.7029